

Bansho keikaku - planned board

1. Before the lesson: intention and planned final board

Type: launch, diagnostic and building a criterion

Kadai	Without expanding, how can we decide whether an expression is a sum or a product?	Objective	Identify the main operation after reading brackets and order of operations; distinguish the written structure from the value. Reactivate Year 9 prior knowledge and briefly present M/R/H. Skills: calculate, investigate, communicate.
Materials	Marked 01A test scripts; inherited flash 3.3 b)-d) from chapter 01A; course notes handout folded over the answer strip; graded exercise sheet; “Gammes du soir” (evening drills) sheet; diagnostic half-sheet; notebook; timer.	Pupils’ work	Three functions are sought in the actual responses to the same sorting task; a fourth comparison may be proposed explicitly by the teacher and then carries the letter T.
ESS essential function CONT contingent form DECL public prompt. Actual responses are labelled R1, R2, ..., without names or initials. If a function is missing, the teacher asks a follow-up question and writes their own proposal T: no fictitious pupil work is introduced.			

Planned final board at the end of the lesson.

IDEAL BOARD (final state): to project or reproduce

LEFT: the problem and the working agreement

Kadai: “Without expanding, how can we decide whether an expression is a sum or a product?”

Three tracks: M = core track for all; R = test level; H = going further.

Diagnostic: answers handed in before unfolding the answer strip.

Question kept open for S3: a square of side 10 m increases to 12 m; is the area gained equal to 2^2 ?

CENTRE: anonymous responses to sorting task 1.1

1.1: $3x + 2$ sum; $5(x + 4)$ product; $x^2 - 9$ sum; $(2x - 3)^2$ product ($= (2x - 3)(2x - 3)$)

~~“(2x - 3)² is a sum because of the -”~~
 ~~$(2x - 3)^2 = (2x - 3) \times (2x - 3)$: the square is read as a product.~~

~~“ $x(x + 1) + 3(x + 2)$ is a product”: the addition applies to the whole expression.~~

T: $5(x + 4) = 5x + 20$ for all x ; on the left a product, on the right a sum.

RIGHT: what we retain

Main operation: the one that applies to the whole expression, after brackets and order of operations.

$+/- \rightarrow$ sum (of terms)

$\times \rightarrow$ product (of factors)

$E^2 = E \times E$: a square is read as a product.

We classify an expression, not its value.

PRODUCT $\xrightarrow{\text{expand}}$ SUM

SUM $\xrightarrow{\text{factorise}}$ PRODUCT

Physical board rule: problem, selected work, refuted errors, connections and matome remain visible until the final photograph. Only auxiliary calculations may be erased after validation and verbalisation.

2. Storyboard: write, show, connect, keep, erase

Consistent colour code: main writing in base colour; relation, logical connector or invariant in an accent colour; error crossed out but legible. Outside the auxiliary strip, no selected work is erased during the lesson.

Time	Board state	What is written	Teacher talk and activity	Board action / fate
Before entry	T0 - discreet outline	Draw three areas; reserve spaces for R1–Rk, T and the right-hand area.	Prepare the seating plan; do not pre-write the answer.	RESERVE spaces; everything remains available.
0–10	T1 - 01A test feedback	Display an anonymised actual error; locate the breakdown, fix one line and formulate a transferable rule.	“At which line does the reasoning stop being valid? What rule would have prevented this error?”	PHOTOGRAPH or record the correction; erase at 10 min to separate the two chapters.
10–15	T2 - Inherited 01A flash	Reuse the structure of 3.3 b)–d): find k from an effect on areas or volumes; figures possibly changed.	“Which effect is given? Do we need to find k , k^2 or k^3 ?”	MARK immediately then erase; this flash replaces any other fluency drill.
15–17	T3 - Targeted working agreement	Have pupils locate M/R/H and the status of the boxes; read only two sentences of the starting point.	“What is the difference between the core track, the test level and going further?”	KEEP M/R/H in the left-hand corner.
17–27	T4 - Closed diagnostic	Seven minutes alone on a half-sheet, handout folded over the answers; collect, then three minutes of self-checking by unfolding the strip.	“Before marking, which item required a rule you could no longer remember?”	NO visible answer before collection; keep only the numbers of the items to revisit.
27–38	T5 - Investigation on sorting task 1.1	Four minutes alone, three minutes in pairs to justify, then collection of actual responses. Circle the main operation, without expanding.	“Which operation applies to the whole expression, once the order of operations is respected?”	WRITE R1, R2, ..., without identifier; reserve T.
38–46	T6 - Neriage	Compare the internal-sign error, the rewriting of the square, then the sum of products; introduce $5(x + 4) = 5x + 20$ as proposal T.	“Is the – sign the main operation? Must two equal expressions have the same structure?”	KEEP legible error, refutation, connectors and comparison T.
46–49	T7 - Handover of the square problem	Read only the vegetable patch situation and Karim’s claim; announce that the table is to be prepared for S3.	“How can we compare the old area and the new one without deciding straight away?”	KEEP the question, no value or answer formula.
49–55	T8 - Protected matome and copying	Have pupils formulate the main operation, the square convention and the two transformations.	“In $x(x + 1) + 3(x + 2)$, which operation applies to the whole expression?”	FINAL PHOTO: Kadai, R1–Rk, T, relations, matome and vegetable patch question.

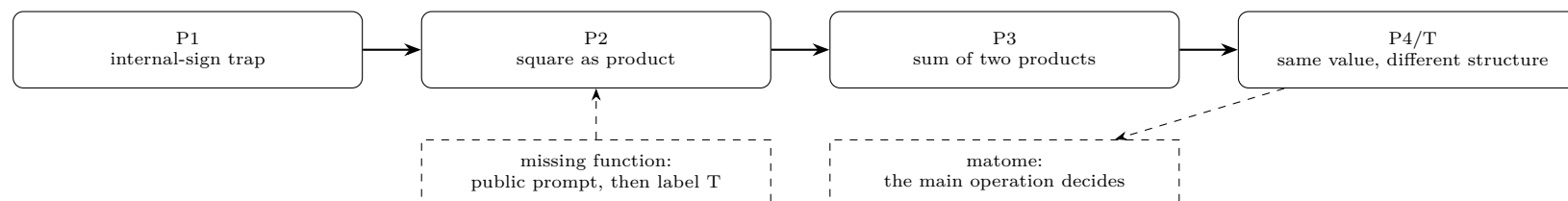
Time-consistency check: the diagnostic is closed before any answer; copying only starts after the comparison. If running late, postpone the vegetable patch launch and set it as preparation for S3; do not shorten the neriage or the matome.

3. Orchestrating the discussion: select, order, connect

ID	Anticipated pupil work	Function in the lesson narrative	Exact questions	Placement and fate
P1	" $(2x - 3)^2$ is a sum because it contains a minus sign."	Distinguish internal signs from the main operation.	"Is the minus sign carried out before or after squaring?"	Centre, block 1; cross out "sum", without erasing the argument.
P2	$(2x - 3)^2 = (2x - 3) \times (2x - 3)$: in this reading, the main operation is a multiplication.	Make the square readable as a product of two identical factors.	"How can we rewrite the square to make the product visible?"	Centre, block 2; keep the rewriting.
P3	" $x(x + 1) + 3(x + 2)$ is a product because it contains multiplications." Then correction: the addition applies to the whole expression.	Check that the criterion applies to the main operation, not to the operations visible inside the terms.	"What do we calculate after the two products: each one separately or their sum?"	Centre, block 3; cross out "product", keep the argument and then its correction.
P4/T	$5(x + 4) = 5x + 20$ for all x : the first expression is a product, the second a sum.	Distinguish the structure of the expression from its value; link to the meaning of expanding.	"Does the distributive property change the values obtained? Does it change the written main operation?"	Centre, block 4; announce "proposal T" and align the two expressions.

Planned selection order. Initial response or productive error → comparison → stabilised procedure → justification or matome. After each contribution: "Who used this idea?" then "Who can suggest a more general justification?"

Essential / contingent. The four functions structure the comparison; the authors, the exact words and the number of actual responses remain contingent. On the board, write R1-Rk without identifier and then fill in their function. Comparison P4 is announced as proposal T if it does not come from the class.



Structured variation. V1 internal – sign; V2 square rewritten as a product; V3 sum in which each term is a product; V4 two equal expressions obtained by the distributive property, but with different structures.

Invariant to have pupils verbalise. After reading brackets and order of operations, the operation that applies to the whole expression is the main operation: addition or subtraction for a sum, multiplication for a product; E^2 is read as $E \times E$.

Scope and boundary. We classify an expression, not the number it represents. Two expressions equal for all values of x can have different structures.

4. Comparing the plan and the board actually built

Observed point	Planned	Actual / evidence collected
Investigation and whole-class discussion time	0–10 test feedback; 10–15 inherited flash; 15–17 working agreement; 17–27 closed diagnostic; 27–38 sorting task; 38–46 neriage; 46–49 vegetable patch launch; 49–55 matome.	N/A
Work that emerged	P1 internal sign/main operation confusion; P2 square rewritten as a product; P3 sum made up of two products; P4/T equality of two expressions with different structures.	N/A
Order of presentation	initial / conflict → comparison → method → justification	N/A
Useful unexpected work	space Rk reserved; specify whether T was needed	N/A
Question that moved things forward	“Which operation applies to the whole expression?”; “How can we rewrite the square?”; “does same value mean same structure?”	N/A
Elements that remained visible	Kadai, R1–Rk, possible proposal T, refutations, relations, matome	N/A
Elements erased and when	01A feedback erased at 10 min after photograph/recording; afterwards, auxiliary calculations only after verbalisation	N/A
Matome formulated by the pupils	The main operation applies to the whole expression; expanding goes from product to sum, factorising from sum to product.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert the photograph here
or indicate its filename

Photograph checklist

- ☐ The problem and the goal are identifiable.
- ☐ R1–Rk are distinguishable; any teacher proposal is labelled T.
- ☐ A refuted error is not confused with the result.
- ☐ The relations and connectors are written.
- ☐ The invariant or condition is visible.
- ☐ The matome is understandable without the teacher's spoken explanation.

Class indicator

Observed work or change of mind: _____

Deferred copying

Actual start time: _____

Local decision

Element to keep: _____

Single adjustment

Element to modify: _____

Bansho keikaku - planned board

Chapter 01B - S1 - Algebraic expressions and notable identities

1. Before the lesson: intention and planned final board

Type: problem-based lesson and proof

Kadai	How many products must be written to expand $(a+b)(c+d)$, and how can this be justified using only the distributive property?	Objective	Establish expanding double brackets (double distributive property) for all real numbers a, b, c, d , then apply it without losing signs. Skills: investigate, reason, calculate.
Materials	Course notes Part 2; exercise sheet 1.3; mini-whiteboards.	Pupils' work	P1–P3 structure the neriage; P4 is another algebraic proof, contingent and compared only if it appears.
ESS essential function CONT contingent form DECL teacher prompt. The responses actually collected are anonymised as R1–Rk; a proposal or step provided by the teacher is labelled T.			

Planned final board at the end of the lesson.

IDEAL BOARD (final state): to project or reproduce

LEFT: problem

Kadai: How many products must be written to expand $(a+b)(c+d)$, and how can this be justified using only the distributive property?

For real numbers a, b, c, d .

Do a recipe or four arrows already provide a proof?

CENTRE: pupils' work

R1 or T: $(a+b)(c+d) = ac + bd$
 $a = b = c = d = 1$: $4 \neq 2$; the general equality is disproved.

R2: four arrows: $ac + ad + bc + bd$.

R3 or T, proof: $k = a + b$;
 $(a+b)(c+d) = k(c+d) = kc + kd$
 $= (a+b)c + (a+b)d = ac + bc + ad + bd$
 $= ac + ad + bc + bd$ (commutativity of addition)

RIGHT: summary (written summary)

For all real numbers a, b, c, d :
 $(a+b)(c+d) = ac + ad + bc + bd$.

Method: four products with the signs of their two factors, then simplify.

Ex. $(x-3)(2x+5)$
 $= 2x^2 + 5x - 6x - 15 = 2x^2 - x - 15$.
For $x = 1$, both expressions equal -14 :
spot check, not a proof.

Physical board rule: problem, selected work, refuted errors, connections and matome remain visible until the final photograph. Only auxiliary calculations may be erased after validation and verbalisation.

2. Storyboard: write, show, connect, keep, erase

Consistent colour code: main writing in base colour; relation, logical connector or invariant in an accent colour; error crossed out but legible. Outside the auxiliary strip, no selected work is erased during the lesson.

Time	Board state	What is written	Teacher talk and activity	Board action / fate
Before entry	T0 - discreet outline	Draw three areas; reserve P1–P3, a contingent space P4 and the right-hand area.	Prepare the seating plan; do not pre-write the answer.	RESERVE spaces; everything remains available.
0–8	T1 - Fluency and 01A code	One question on the dominant R/A/K/D/U code that is still fragile replaces one of the three expansions in 1.2 a; complete the two remaining expansions and the first two opposites in 1.2 b. Keep $-(5 - 4x) = -1(5 - 4x) = -5 + 4x$.	“Which code is being revisited? What does the number -1 multiply?”	NOTE the code and the observed result; erase the other calculations after marking.
8–19	T2 - Kadai and investigation	Write the Kadai. Individual investigation 4 min then pairs 4 min; collect and code the responses without giving the arrows.	“Is your line a conjecture, a recipe or a proof?”	RESERVE the order R1–Rk after circulating.
19–34	T3 - Neriage and proof	Disprove $ac + bd$ if this equality appears or if T puts it to the test; compare the four arrows with the proof using $k = a + b$; write the final reordering.	“Which property justifies each equality? What does testing at 1 prove?”	KEEP counterexample, procedure, three uses of the distributive property and commutativity.
34–47	T4 - Signed variation: 1.3 a, b, d	Have pupils work on a, b and d; c is reserved for fast finishers. Require all four products before simplifying and check b or d at $x = 1$.	“Does the sign in -3 belong to the term? Is testing at 1 enough to prove it?”	KEEP a model answer; explicitly write “check, not proof”.
47–53	T5 - Matome formulated then copied	Have pupils formulate the domain, the formula, the reason for the four products and the order “expand then simplify”.	“What sentence explains the rule without mentioning arrows?”	KEEP the path to the matome; deferred copying.
53–55	T6 - Exit check	Mini-whiteboard: expand $(x - 4)(x + 1)$ showing all four products.	Observe result and intermediate line; note any need for follow-up.	FINAL PHOTO before any board reset.

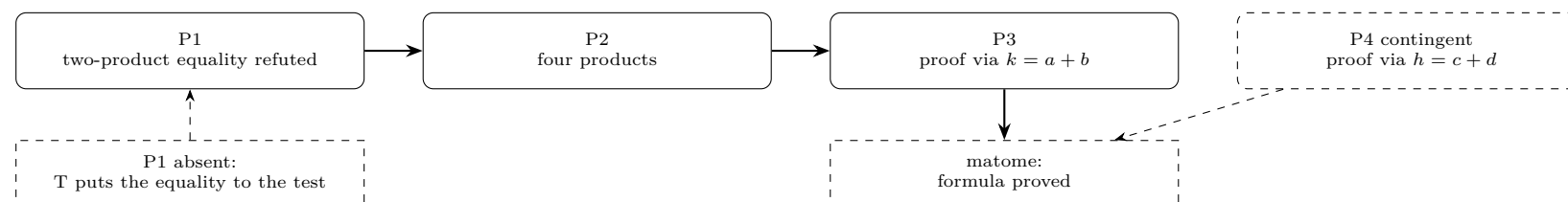
Time-consistency check: investigation, P1–P3 comparison, proof, six minutes of matome and exit check are non-negotiable. If time is short, drop P4 first and then practice item d; never fabricate a missing pupil response.

3. Orchestrating the discussion: select, order, connect

ID	Anticipated pupil work	Function in the lesson narrative	Exact questions	Placement and fate
P1	$(a + b)(c + d) = ac + bd$: two products only. For $a = b = c = d = 1$, the two sides equal 4 and 2.	Distinguish disproving a general equality from proving the correct formula.	“What can we conclude from this single counterexample? Does it already give us the correct formula?”	Centre, block 1; if the error does not appear, T proposes it as a claim to test, without attributing it to a pupil.
P2	Four arrows and $ac + ad + bc + bd$, without written justification.	Provide a plausible result and procedure, still distinct from a proof.	“Which known property justifies each of the four arrows?”	Centre, block 2; keep the procedure and then connect it to the lines of P3.
P3	Let $k = a + b$; apply the distributive property three times; swap bc and ad by commutativity of addition.	Prove the formula for all real numbers and also justify the standard order of the terms.	“On which lines do we use the distributive property? Why can we go from $ac + bc + ad + bd$ to $ac + ad + bc + bd$?”	Centre, block 3; P3 is essential. If T supplies k , label it T; the steps given by the class remain R.
P4	Let $h = c + d$: $(a + b)h = ah + bh = a(c + d) + b(c + d) = ac + ad + bc + bd$.	Compare a second algebraic proof valid for all real numbers; make the symmetry of the two factors visible.	“What happens if we name the second bracket rather than the first?”	Contingent; only write it up if it is actually produced or reconstructed after this prompt.

Planned selection order. Refuted general equality → correct but unproven result → proof derived from known facts → possible alternative proof → matome. After each contribution: “What does this piece of work establish exactly?”

Essential / contingent. Functions P1–P3 are essential; P4 is contingent. The authors and wordings remain unknown before the lesson. Record R1–Rk for the observed answers and T for any teacher input, then fill in the R/P correspondence.



Structured variation. V1 false equality and counterexample; V2 correct result without proof; V3 proof via the first block; V4 possible proof via the second block; then sign variations on 1.3. Each change serves a distinct function.

Invariant to have pupils verbalise. For all real numbers a, b, c, d , $(a + b)(c + d) = ac + ad + bc + bd$ follows from three applications of the distributive property, and possibly a reordering of the terms of a sum. Each product keeps the sign of its two factors before simplifying.

Scope of validity. The “four products” apply to two sums of two terms. With a subtraction, the negative term is treated along with its sign. A numerical check can refute a general equality, but never proves it.

4. Comparing the plan with the board actually built

Observed point	Planned	Actual / evidence collected
Individual work and whole-class discussion time	0–8 fluency; 8–19 individual work; 19–34 neriage and proof; 34–47 variation; 47–53 matome; 53–55 exit.	N/A
Pupil work that emerged	P1 refuted equality; P2 four arrows; P3 proof via $k = a + b$; P4 possible proof via $h = c + d$.	N/A
Order of presentation	refutation → unproven result → proof → possible alternative proof	N/A
Useful unexpected contribution	contingent space reserved; answer recorded as R, never fabricated	N/A
Question that moved the lesson forward	“What does testing with 1 prove?”; “Which property justifies this equality?”	N/A
Elements kept visible	Kadai, R1–Rk, possible T, links, domain, proof and matome	N/A
Elements erased and when	auxiliary calculations only, after validation and verbalisation	N/A
Matome formulated by the pupils	For two sums of two terms, each term of the first factor multiplies each term of the second; three applications of the distributive property prove it for all real numbers.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert photograph here
or state its filename

Photograph checklist

- ☐ The problem and the goal are identifiable.
- ☐ R1–Rk are distinguishable; any teacher input is marked T.
- ☐ A refuted error is not confused with the result.
- ☐ Links and logical connectors are written out.
- ☐ The invariant or condition is visible.
- ☐ The matome is understandable without the teacher’s spoken commentary.

Class indicator	Observed work or change of mind: _____	Deferred copying	Actual start time: _____
Local decision	Element to keep: _____	Single adjustment	Element to modify: _____

Bansho keikaku - planned board

1. Before the lesson: intention and planned final board

Type: structured practice, comparison of strategies and transfer

Kadai	How can we expand $C = 3x(x - 3) - (x + 7)(4x - 2)$ without losing the sign in front of the second product, and then check the result?	Objective	Subtract an expanded product accurately; compare a complete error with a safe method and, if it arises, with another correct path; distinguish between transformation, refutation and check. Then translate a calculation program in x .
Equipment	Exercise sheet 1.3, 1.4 c) and 5.3; mini-whiteboards; timer.	Outputs	P1 carries the error: complete observed answer or, failing that, equality T to test; P2 is a complete safe method; P3 a possible alternative correct path; P4 compares by substitution.
ESS essential function CONT contingent form DECL possible teacher input. Observed pupil outputs are labelled R1–Rk, without names; any teacher input is marked T. The R/P mapping is filled in after circulating.			

Planned final board at the end of the slot.

IDEAL BOARD (final state): to project or reproduce

LEFT: Kadai and compared answers

Kadai: How can we expand $C = 3x(x - 3) - (x + 7)(4x - 2)$ without losing the sign in front of the second product, and then check the result?

R1 or T, error:

$$3x^2 - 9x - 4x^2 + 26x - 14 \\ = -x^2 + 17x - 14.$$

R2, safety brackets:

$$3x^2 - 9x - (4x^2 + 26x - 14) \\ = 3x^2 - 9x - 4x^2 - 26x + 14 \\ = -x^2 - 35x + 14.$$

Possible R3:

$$3x(x - 3) + (-x - 7)(4x - 2) \\ = -x^2 - 35x + 14.$$

$x = 1$: initial -22 ; R1 gives 2 ; R2 gives -22 .

CENTRE: varied proof then program

Proof from S1 restated, then:

$$k = a + b; \quad k(c - d) = kc - kd \\ = (a + b)c - (a + b)d \\ = ac + bc - (ad + bd) \\ = ac - ad + bc - bd.$$

Same distributive property; the subtracted group remains visible.

Program 5.3:

$$P(1) = 3; \quad P(2) = 5. \\ P(x) = (x + 1)^2 - x^2 \\ = (x + 1)(x + 1) - x^2 \\ = x^2 + x + x + 1 - x^2 = 2x + 1.$$

If x is an integer, $2x + 1$ is odd.

Getting 15: $x = 7$ (if time permits).

RIGHT: matome

Subtracting a group:

$$A - B = A + (-1)B.$$

I expand B inside brackets, then distribute -1 to each of its terms; I simplify at the end.

A conflicting value refutes a general equality. A matching value only checks it; the proof comes from justified equalities.

A calculation program translates into an expression $P(x)$. Trials suggest a result; algebra establishes it for all numbers in the domain.

Board rule: problem, selected outputs, refuted errors, links and matome remain visible until the final photograph. Only auxiliary calculations may be erased after validation and verbalisation.

2. Storyboard: write, show, link, keep, erase

Consistent colour code: main writing in base colour; relation, logical connector or invariant in an accent colour; error crossed out but legible. Outside the auxiliary strip, no selected output is erased during the lesson.

Time	Board state	What is written	Speech and activity	Physical action / fate
Before entry	T0 - discreet outline	Draw the zones; reserve space for three possible answers, a check strip and the matome on the right.	Prepare the classroom layout; pre-write neither pupil outputs nor results.	RESERVE spaces; everything remains available.
0–8	T1 - Fluency and 01A code	One question from the second R/A/K/D/U code still fragile replaces 1.3 e); do f), g), h) without a calculator, then mark selectively.	“Which code is revisited? In f), what does the -1 multiply? In h), which terms cancel out?”	RECORD the code and observed result; erase calculations after validation, except one line linked to the Kadai.
8–17	T2 - Proof from S1 restated and varied	A volunteer restates the proof using $k = a + b$; the class checks each equality. Then rebuild $k(c - d) = kc - kd$ down to $ac - ad + bc - bd$.	“Where must a group be kept visible? Which property justifies each step?”	KEEP the varied proof in the centre; oral comments never replace an equality.
17–33	T3 - Kadai: individual work then neriage	Kadai; individual work 4 min, pairs 3 min, discussion 9 min. Compare a complete erroneous answer, the bracket method, a possible alternative method and the check at $x = 1$.	“Do these two expressions transform the same group? What can we conclude from a discrepancy at $x = 1$?”	CROSS OUT the error without erasing it; keep the safe chain, the possible alternative method and the logical status of the test.
33–44	T4 - Program 5.3: from particular to general	Calculate for 1 and 2, translate into $P(x)$, then expand by expanding double brackets. Write the condition “if x is an integer” before concluding odd.	“Do the two trials prove the formula $2x + 1$? Which line holds for all x ?”	KEEP program, two trials, expression and simplification; mark the general case with a connector.
44–52	T5 - Matome formulated then copied	Prompt pupils to state $A - B = A + (-1)B$, the safe procedure and the two logical statuses of a test. Link trials and algebra for the program.	“Which sentence accurately distinguishes between refuting, checking and proving?”	KEEP the path to the matome; delayed copying for eight minutes.
52–55	T6 - Exit check	Expand $4(3x - 2) - (2x - 5)$ showing the intermediate brackets. Expected: $10x - 3$.	Observe the scope of the minus sign and the simplification separately; note pupils needing follow-up.	FINAL PHOTO before wiping the board.

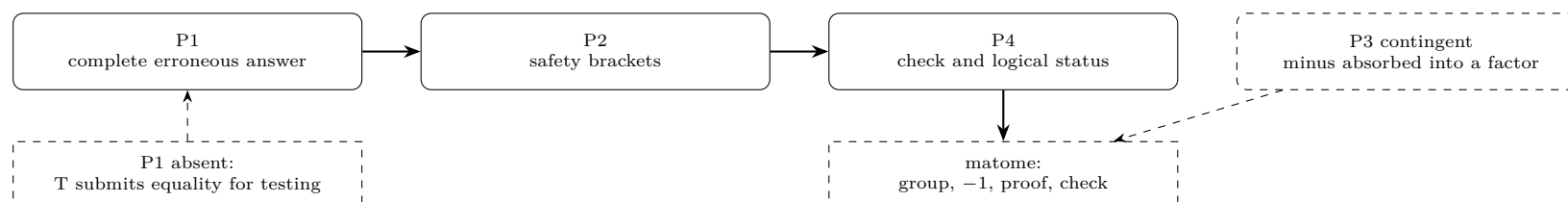
Time-management check: if short of time, drop R3 first, then the equation “getting 15”. The checked restatement, Kadai investigation, bracket method, matome and exit check remain incompressible. The program may stop at $P(x) = 2x + 1$.

3. Managing the discussion: select, order, connect

ID	Anticipated output	Role in the lesson narrative	Exact prompts	Placement and fate
P1	Complete erroneous answer: $3x^2 - 9x - 4x^2 + 26x - 14 = -x^2 + 17x - 14$. Only the first term of the group has changed sign.	Highlight the scope error without reducing it to a mere local slip.	"Which whole object does the minus sign apply to in the starting expression? Does your line preserve that scope?"	Left, block 1; cross out after refutation. If it does not appear, T introduces it as an equality to be tested.
P2	Complete safe method: $3x^2 - 9x - (4x^2 + 26x - 14) = 3x^2 - 9x - 4x^2 - 26x + 14 = -x^2 - 35x + 14$.	Physically preserve the group, then distribute -1 before simplifying.	"Why are the brackets still needed on the first line? Which property justifies the next line?"	Left, block 2; essential, kept until the matome.
P3	Alternative method: $C = 3x(x - 3) + (-x - 7)(4x - 2) = -x^2 - 35x + 14$. The minus sign is absorbed into the first factor of the second product.	Compare two correct strategies and spot the same invariant under a different form.	"Where did the factor -1 go? Does this method change the value of the product?"	Centre, contingent; only write it up if it is produced or reconstructed after a prompt to the class.
P4	For $x = 1$: initial expression $= -22$; P1 gives 2; P2 gives -22 .	Distinguish the logical force of a discrepancy from the weakness of an isolated match.	"What does $2 \neq -22$ refute? What does $-22 = -22$ prove, or fail to prove?"	Check strip; observed answer or explicitly marked T prompt. Keep both conclusions.

Planned selection order. P1 complete erroneous answer → P2 complete safe solution → P4 comparative check → P3 possible alternative solution → matome. After each contribution: "What exactly does this output establish?"

Essential / contingent. P1, P2 and P4 serve essential functions; P3 is contingent. Authors and wording remain unknown before the lesson. Label R1–Rk without pupil identifiers, T for any teacher input, then fill in R/P.



Structured variation. V1 wrong scope; V2 preserved group; V3 sign absorbed into a factor; V4 check on the same candidates. The expression and goal remain invariant; only the strategies vary.

Invariant to verbalise. Subtracting a group amounts to multiplying it by the factor -1 . Distributing this factor comes before any simplification, whichever form is chosen.

Scope of validity. A conflicting substitution refutes the tested equality. A matching substitution checks one value only; validity for all real numbers rests on the chain of equalities.

4. Comparing the plan with the board actually built

Observed item	Planned	Actual / evidence gathered
Investigation and whole-class discussion time	0–8 fluency; 8–17 varied proof; 17–33 investigation and neriage; 33–44 program; 44–52 matome; 52–55 exit check.	N/A
Outputs observed	P1 complete erroneous answer; P2 bracket solution; P3 possible alternative solution; P4 comparative check.	N/A
Order of presentation	complete error → safe solution → check → possible alternative method → matome	N/A
Useful unexpected output	contingent space reserved; answer labelled R, never fabricated	N/A
Question that moved thinking forward	“Which whole object does the minus apply to?”; “What does the discrepancy refute? What does the match prove?”	N/A
Elements kept visible	Kadai, R1–Rk, possible T, varied proof, checks, program and matome	N/A
Elements erased and when	auxiliary calculations only, after validation and verbalisation	N/A
Matome formulated by pupils	Subtracting a group amounts to multiplying each of its terms by -1 . A numerical discrepancy refutes; an isolated match checks without proving.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert photograph here
or state its file name

Photograph checklist

- ☐ The problem and goal are identifiable.
- ☐ R1–Rk are distinguishable; any teacher input is marked T.
- ☐ A refuted error cannot be mistaken for the result.
- ☐ Links and connectors are written down.
- ☐ The invariant or condition is visible.
- ☐ The matome can be understood without the teacher's spoken commentary.

Class indicator

Output or change of mind observed: _____

Delayed copying

Actual start time: _____

Local decision

Element to keep: _____

Single adjustment

Element to modify: _____

Bansho keikaku - planned board

1. Before the lesson: intention and planned final board

Type: one problem, multiple representations, neriage

Kadai	The table and S2 suggest $(a+b)^2 = a^2 + 2ab + b^2$. Why does the term $2ab$ appear, and how can we prove this equality for all real numbers a, b ?	Objective	Distinguish between data, conjecture and proof; connect two area readings to expanding double brackets; specify the domain of each justification. Skills: problem-solving, representing, reasoning, communicating.
Equipment	Course notes, introductory activity and part 3; result from S2 $(x+1)^2 = x^2 + 2x + 1$; exercise sheet 2.1 a), b), h); blank squares; mini-whiteboards.	Outputs	P1 numerical pattern; P2 four-piece dissection; P3 two-strip reading, contingent; P4 algebraic proof for all real numbers.
ESS essential function CONT contingent form DECL teacher input, explicitly marked T. Actually observed pupil answers are labelled R1–Rk; no answer or pupil initial is pre-assigned.			

Planned final board at the end of the slot.

IDEAL BOARD (final state): to project or reproduce

LEFT: data and Kadai

Prepared data:

a	b	$(a+b)^2 - (a^2 + b^2)$	$2ab$
10	2	40	40
3	4	24	24
5	1	10	10
7	10	140	140

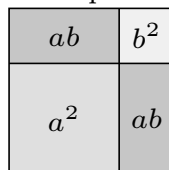
S2: $(x+1)^2 = x^2 + 2x + 1$.

These cases suggest; they do not prove.

Kadai: The table and S2 suggest $(a+b)^2 = a^2 + 2ab + b^2$. Why does the term $2ab$ appear, and how can we prove this equality for all real numbers a, b ?

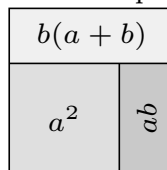
CENTRE: two readings of the same square

Four pieces



$$a^2 + ab + ab + b^2$$

Two strips



$$a^2 + ab + b(a+b)$$

Same area, hence equal expressions. For non-negative lengths, each gives $(a+b)^2 = a^2 + 2ab + b^2$.

RIGHT: general proof and matome

For all real numbers a, b :

$$\begin{aligned} (a+b)^2 &= (a+b)(a+b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + 2ab + b^2. \end{aligned}$$

Matome:

$$(a+b)^2 = a^2 + 2ab + b^2$$

The term $2ab$ comes from two cross-products, visible as two rectangles of area ab .

An identity is an equality that holds true for all permitted values of the letters.

The statement “for all a, b , $(a+b)^2 = a^2 + b^2$ ” is refuted by $(3, 4)$: $49 \neq 25$.

Board rule: data, Kadai, selected outputs, comparison arrows, algebraic proof and matome remain visible together. Only auxiliary trials and the practice strip may be erased once they have served their purpose.

2. Storyboard: write, show, link, keep, erase

Consistent colour code: main writing in base colour; relation, logical connector or invariant in an accent colour; error crossed out but legible. Outside the auxiliary strip, no selected output is erased during the lesson.

Time	Board state	What is written	Speech and activity	Physical action / fate
Before entry	T0 - discreet outline	Draw three zones; reserve space for data, two area readings and the general proof.	Prepare the classroom layout; pre-write neither pupil outputs nor the matome.	RESERVE spaces; everything remains available.
0–7	T1 - reactivated data	Copy the four differences from the table; recall the S2 case $(x + 1)^2 = x^2 + 2x + 1$; gather conjectures R1–Rk, then write up the Kadai.	“What do these four rows allow us to state? What do they not yet allow us to state?”	KEEP data, “conjecture” label and Kadai. Do not validate a formula from trials alone.
7–18	T2 - individual then paired investigation	Hand out a blank square of side $a + b$: pupils investigate individually for 5 min, then compare for 6 min. Ask for a visual justification and, if possible, a figure-free route.	“Without choosing numerical values, how can we account for the exact total area? Can you see the large square in another way?”	CIRCULATE; note actual answers and select complete lines of reasoning.
18–35	T3 - neriage and general proof	Sequence P1, P2, P3 if present, then P4. Link the terms to the pieces; compare $a^2 + ab + ab + b^2$ and $a^2 + ab + b(a + b)$; finally write the algebraic chain.	“What varies between the diagrams? What stays the same? Which proof also covers negative values?”	KEEP selected outputs side by side; P3 is omitted from the board if it does not arise.
35–44	T4 - protected matome and copying	Prompt pupils to state the domain of each proof, the origin of $2ab$ and the definition of an identity; then display the written summary to copy word for word.	“Which sentence distinguishes the roles of the table, the diagram and algebra?”	ALLOW copying only after the comparison; keep the proofs visible.
44–52	T5 - targeted practice	Do only exercise sheet 2.1 a), b), h). For each square, identify the two building blocks before substituting into the S3 formula.	“In $(x + 3)^2$, which blocks play the roles of a and b ? Where do the two cross-products appear?”	USE the bottom strip; keep a single worked example or erase it after validation.
52–55	T6 - exit check	Mini-whiteboard: $(2x + 3)^2$. Require $a = 2x$, $b = 3$, then $4x^2 + 12x + 9$.	“Which term lets you check that the double product has not been forgotten?”	PHOTOGRAPH the main board while collecting the mini-whiteboards.

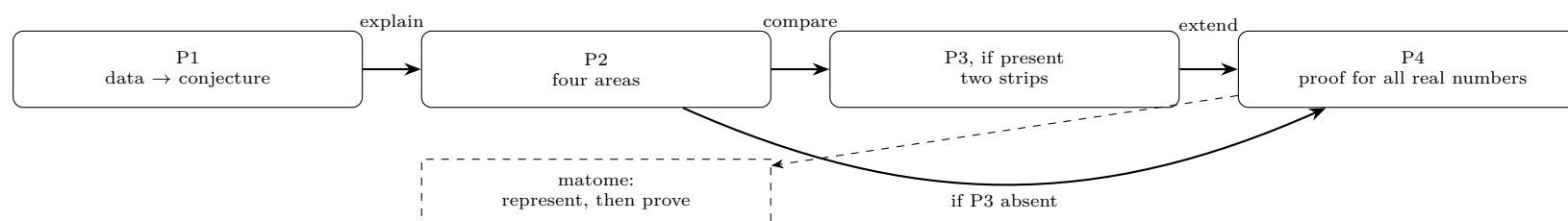
Fallback rule: drop P3 first, then reduce practice to 2.1 a) and h). The starting data, one visual justification, the algebraic proof, nine minutes for the matome and the exit check are incompressible. The other two identities are studied in S4, as planned.

3. Managing the discussion: select, order, connect

ID	Anticipated output	Role in the lesson narrative	Exact prompts	Placement and fate
P1	All four differences equal $2ab$; the S2 case corresponds to $b = 1$. The pupil states $(a + b)^2 = a^2 + 2ab + b^2$ as a conjecture.	Let the formula emerge from the data already produced, while stressing that a finite list does not prove a universal statement.	"What stays invariant across the four rows? How does the case $b = 1$ from S2 support the same conjecture?"	Left, below the data; keep with the note "suggests, does not prove".
P2	Four-piece dissection: $a^2 + ab + ab + b^2$. Actually observed output or collective construction from a blank figure.	Explain the coefficient 2 geometrically via two distinct rectangles of area ab .	"Which side lengths justify each area? Do the four pieces cover the square without gaps or overlaps?"	Centre left; link each term to its piece and keep. If the teacher provides the dissection, label it T.
P3	Two-strip reading: $a^2 + ab + b(a + b)$, then expanded to $a^2 + 2ab + b^2$.	Compare two exact ways of partitioning the same surface and make the equality of the expressions visible.	"Where does the strip of area ab end? Why is the top strip equal to $b(a + b)$?"	Centre right; CONT only write it up if it arises after a prompt to the class.
P4	$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$.	Provide the general proof and extend the result from non-negative lengths to all real numbers.	"Which property justifies the four products? Why is $ab + ba = 2ab$? Which domain does this chain cover?"	Right, before the matome; reconstruct collectively if no pupil's work is complete, without attributing the chain to a fictitious pupil.

Planned selection order. P1 data and conjecture → P2 four-piece reading → P3 alternative reading if present → P4 algebraic proof → matome. After each contribution: have another pupil rephrase it and ask what the answer adds to the previous one.

Essential / contingent. P1, one complete area reading and P4 are essential; P3 is contingent. A missing output is never replaced by a fictitious pupil's work: a collective construction or a teacher input T is labelled as such. Actual authors are noted only after observation.



Structured variation. The number of cases, the dissection and the register change; the total area and the two cross-products remain. Thus the neriage does not merely juxtapose three answers: it makes their relationships explicit.

Invariant to verbalise. The square of a sum is the product of two identical sums. The two cross-products ab and ba are equal; their sum is $2ab$.

Scope of validity. Areas justify the formula when $a, b \geq 0$; the algebraic chain proves it for all real numbers. The equality $(a + b)^2 = a^2 + b^2$ is not an identity: the pair $(3, 4)$ refutes the universal statement, even though some pairs, for instance $b = 0$, satisfy it.

4. Comparing the plan with the board actually built

Observed item	Planned	Actual / evidence gathered
Investigation and whole-class discussion time	0–7 data; 7–18 investigation; 18–35 neriage; 35–44 matome; 44–52 practice; 52–55 exit check.	N/A
Outputs observed	P1 conjecture from data; P2 four-piece reading; P3 possible strips; P4 complete or reconstructed algebraic proof.	N/A
Order of presentation	data → first area → possible second area → proof for all real numbers → matome	N/A
Useful unexpected output	note its expression, its domain and its link with P2 or P4	N/A
Question that moved thinking forward	“What do the examples suggest without proving?”; “What stays identical in both dissections?”; “Which proof covers negative numbers?”	N/A
Elements kept visible	data, Kadai, selected R1–Rk, links across registers, algebraic proof and matome	N/A
Elements erased and when	auxiliary trials and practice only, after comparison; specify what had fulfilled its role	N/A
Matome formulated by pupils	Data lead to a conjecture, areas explain $2ab$ for lengths, and expanding double brackets proves the identity for all real numbers.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert photograph here
or state its file name

Photograph checklist

- ☐ The problem and goal are identifiable.
- ☐ The actually selected answers R1–Rk are identifiable.
- ☐ Any teacher input T is not attributed to a pupil.
- ☐ Links and connectors are written down.
- ☐ Both geometric and algebraic domains are visible.
- ☐ The matome can be understood without the teacher’s spoken commentary.

Class indicator	Output or change of mind observed: _____	Delayed copying	Actual start time: _____
Local decision	Element to keep: _____	Single adjustment	Element to modify: _____

Bansho keikaku - planned board

Chapter 01B - S4 - Algebraic expressions and notable identities

1. Before the lesson: intention and planned final board

Type: one master diagram, two regions, comparison and generalisation

Kadai	How can the same partitioned square explain the two missing identities, and what proof makes them true for all real numbers?	Objective	Construct $(a - b)^2$ and $a^2 - b^2$ from two regions of the same diagram; do not confuse their forms; distinguish between the geometric domain and the algebraic domain. Skills: investigating, representing, reasoning, communicating.
Equipment	Two unlabelled copies of the partitioned square; cut-out pieces; course notes closed then part 3; exercise sheet 2.4 b), f), g); mini-whiteboards.	Outputs	P1 partitioning into four pieces; P2 reading by strips, contingent; P3 cutting the L-shape and reassembling; P4 expanding double brackets for all real numbers.
ESS essential function CONT contingent form DECL teacher input, explicitly marked T. Actually observed answers are noted R1–Rk; no pupil answer or initial is pre-assigned.			

Planned final board at the end of the lesson.

IDEAL BOARD (final state): to project or reproduce

LEFT: master diagram and Kadai

 $0 < b < a$ for the figure.

$b(a-b)$	b^2
$(a-b)^2$	$b(a-b)$

Kadai: How can the same partitioned square explain the two missing identities, and what proof makes them true for all real numbers?

CENTRE: two regions of the same square

P1, inner square:

$$a^2 = (a - b)^2 + 2b(a - b) + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2.$$

P2 if it appears, strips:

$$(a - b)^2 = a^2 - ab - ab + b^2.$$

P3, the L-shape without b^2 :

$$a^2 - b^2 = a(a - b) + b(a - b)$$

$$= (a + b)(a - b).$$

RIGHT: general proof and matome

P4, all real numbers:

$$(a - b)^2 = (a - b)(a - b)$$

$$= a^2 - ab - ba + b^2 = a^2 - 2ab + b^2,$$

$$(a + b)(a - b) = a^2 - ab + ba - b^2 = a^2 - b^2.$$

Matome:

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

Square of a difference $\neq$ difference of two squares.

Logical contrast: b) false; f) false: the pair (3, 4) refutes “for all”; g) true because $b = 0$ is an example.

Board rule: problem, selected outputs, refuted errors, connections and matome remain visible until the final photograph. Only auxiliary calculations may be erased after validation and verbalisation.

2. Storyboard: write, show, connect, keep, erase

Consistent colour code: main writing in base colour; relationship, logical connector or invariant in an accent colour; error crossed out but legible. Outside the auxiliary strip, no selected output is erased during the lesson.

Time	Board state	What is written	Speech and activity	Board action / status
Before entry	T0 - discreet outline	Draw three zones; reserve space for the master diagram, branches P1–P3 and P4.	Prepare the unlabelled sheets and seating plan; do not pre-write any output.	RESERVE spaces; everything remains available.
0–6	T1 - S3 prompt and Kadai	Check $(x + 10)^2$ on mini-whiteboards; write $(a - b)^2$ and $a^2 - b^2$ without expanding; collect R1–Rk, then the Kadai.	“Do these two expressions mean the same thing? What can we predict, and what remains to be proved?”	KEEP both forms and the Kadai; erase only the warm-up calculation.
6–18	T2 - individual then pair work	Hand out two copies of the same square. Ask for lengths, areas, chains of equalities and, if possible, a proof without a diagram.	“Which area are you calculating? What exactly are you cutting out or moving?”	CIRCULATE; note actual answers and select complete lines of reasoning.
18–34	T3 - neriage then domain	Sequence P1, P2 if it appears, P3, then P4. Connect each expression to a region and compare geometry and algebra.	“Why do we add b^2 back? What are the dimensions of the two pieces of the L-shape? Which proof covers negative values?”	KEEP P1, P3 and P4 side by side; mark any teacher input with T.
34–42	T4 - protected matome	Have pupils state the difference between the two structures, then display the written summary.	“Which sentence now prevents us from confusing $(a - b)^2$ and $a^2 - b^2$?”	ALLOW copying after comparison; diagram and proofs remain visible.
42–50	T5 - votes 2.4 b, f, g	Vote, justify and revote on three statements only; keep counter-examples and supporting example.	“Why does $b = 0$ prove ‘there exists’, but not ‘for all’?”	USE the bottom strip; keep quantifiers and justifications.
50–55	T6 - contrasting exit ticket	Mini-whiteboard: $(3x - 2)^2$ and $(3x + 2)(3x - 2)$, structure named before calculating.	“Which term immediately distinguishes the two results?”	PHOTOGRAPH the main board while collecting materials.

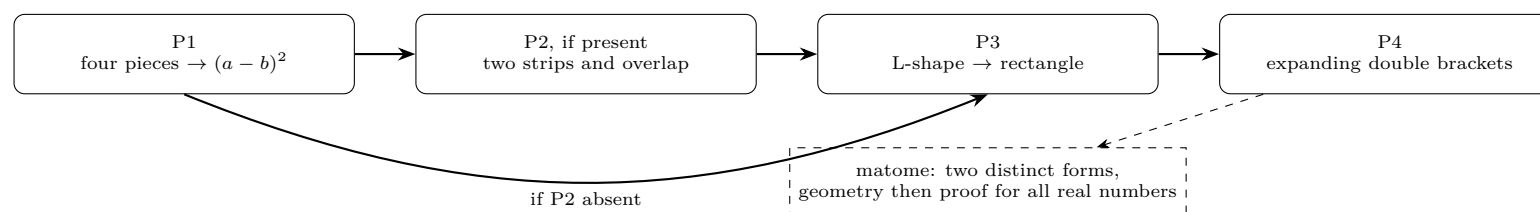
Fallback rule: drop P2 first, then limit the votes to the pair f)–g). P1, P3, both P4 chains, the matome and at least one exit expression are non-negotiable.

3. Managing the discussion: select, sequence, connect

ID	Anticipated output	Role in the narrative	Exact questions	Placement and status
P1	$a^2 = (a - b)^2 + 2b(a - b) + b^2$, then $(a - b)^2 = a^2 - 2ab + b^2$.	Read the inner square as one of the four pieces and explain each term.	"Which four areas make up a^2 ? How do we distribute the minus sign when isolating the square?"	Centre left; essential, keep with the lengths.
P2	$(a - b)^2 = a^2 - ab - ab + b^2$.	Give a second reading of the same square: two strips removed, overlap added back.	"Where is the b^2 that was removed twice? Why must we add it back once?"	Centre, CONT ; write up only if this reading emerges after a whole-class prompt.
P3	$a^2 - b^2 = a(a - b) + b(a - b) = (a + b)(a - b)$.	Read the L-shape as two rectangles, then as a single reassembled rectangle; bring out the factorisation.	"What are the exact dimensions of the two pieces? Which length becomes $a + b$ after re-assembling?"	Centre right; essential, build with the class and mark T if needed.
P4	Two chains of expanding double brackets establish the identities for all real numbers.	Extend the results beyond lengths $0 < b < a$ and distinguish between an explanatory proof and a general proof.	"Which cross-products add together in the square? Which ones cancel out in the product of a sum and a difference?"	Right, before the matome; re-construct as a class and mark T if no pupil's work is complete.

Planned selection order. P1 partitioning → P2 if it appears → P3 cut-and-reassemble → P4 general proof → matome. After each contribution, have pupils restate which area is calculated and what the answer adds.

Essential / contingent. P1, P3 and P4 are necessary; P2 is contingent. A missing approach is never replaced by fictional pupil work: any construction by the teacher is marked T, and only actual answers are marked R1–Rk.



Structured variation. Same square and same lengths; the target region changes: inner square, strips, then L-shape. The register then shifts: area to explain, expanding double brackets to generalise.

Invariant to verbalise. An area does not change by partitioning or by cutting and reassembling; the two cross-products are equal in the square and cancel out in the product of a sum and a difference.

Scope of validity. The diagrams represent $0 < b < a$; P4 proves the identities for all real numbers. In 2.4, one counterexample refutes “for all” and one supporting example suffices for “there exists”.

4. Comparing the plan with the board actually built

Observed item	Planned	Actual / evidence collected
Investigation and whole-class discussion time	0–6 warm-up; 6–18 investigation; 18–34 neriage; 34–42 matome; 42–50 votes; 50–55 exit ticket.	N/A
Outputs that emerged	P1 partitioning the square; P2 possible strips; P3 cutting and reassembling the L-shape; P4 complete or reconstructed algebraic proof.	N/A
Order of presentation	inner square → possible alternative reading → L-shape → proof for all real numbers → matome	N/A
Useful unexpected output	note its region, expression, domain and link with P1, P3 or P4	N/A
Question that moved thinking forward	“Which area are you calculating?”; “Why add b^2 back?”; “Which proof covers negative values?”	N/A
Elements kept visible	master diagram, Kadai, selected R1–Rk, connections between regions, P4 and matome	N/A
Elements erased and when	S3 warm-up and auxiliary trials, only after they fulfilled their role	N/A
Matome stated by pupils	Square of a difference and difference of two squares are distinct; areas explain, algebra proves for all real numbers.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert photograph here
or state its filename

Photograph checklist

- ☐ The problem and the goal are identifiable.
- ☐ The actually selected answers R1–Rk are identifiable.
- ☐ Any teacher input T is not attributed to a pupil.
- ☐ Relationships and connectors are written down.
- ☐ Both geometric and algebraic domains are visible.
- ☐ The matome is understandable without the teacher’s spoken commentary.

Class indicator	Observed output or change of mind: _____	Deferred copying	Actual start time: _____
Local decision	Element to keep: _____	Single adjustment	Element to modify: _____

Bansho keikaku - planned board

1. Before the lesson: intention and planned final board

Type: fixed fluency, tool-choice problem, cost comparison and transfer

Kadai	Is a notable identity always the best tool whenever it can be used? How do we choose a transformation that genuinely simplifies the calculation?	Objective	Distinguish between a valid transformation and an economical transformation; choose between a notable identity and the distributive property; check the plausibility of a result.
Equipment	Exercise sheet 4.1; cards $A = 10,1 \times 9,9$ and $B = 38 \times 998$; 4.4 d; 4.5 a, c; timer; mini-whiteboards.	Outputs	P1 simple midpoint for A ; P2 valid but costly identity for B ; P3 economical distributive property for B ; P4 transfer yielding zero.
ESS essential function	CONT contingent form	DECL teacher input explicitly marked T. Actually observed answers are noted R1–Rk; no pupil work or initial is pre-assigned.	

Planned final board at the end of the lesson.

IDEAL BOARD (final state): to project or reproduce

LEFT: given data, Kadai and first transformation

4.1:

$$n(\geq 6/8) = \underline{\hspace{2cm}}; n(8/8) = \underline{\hspace{2cm}}$$

In S10: same set, no column multiplication, each equality justified, working alone, everyone stops at the same signal.

$$A = 10,1 \times 9,9$$

$$B = 38 \times 998$$

Kadai: Is a notable identity always the best tool whenever it can be used? How do we choose a transformation that genuinely simplifies the calculation?

P1, simple midpoint:

$$\begin{aligned} A &= (10 + 0,1)(10 - 0,1) \\ &= 10^2 - 0,1^2 = 99,99. \end{aligned}$$

CENTRE: compare validity and cost, then transfer

P2 or T, possible identity:

$$\begin{aligned} 518 &= \frac{38 + 998}{2}, 480 = \frac{998 - 38}{2}, \\ B &= (518 - 480)(518 + 480) \\ &= 518^2 - 480^2. \end{aligned}$$

Correct equality, but harder calculation.

P3, useful distributive property:

$$\begin{aligned} B &= 38(1\,000 - 2) \\ &= 38\,000 - 76 = 37\,924. \end{aligned}$$

Valid does not always mean efficient.

P4, transfer:

$$\begin{aligned} &2\,026^2 - 2\,025^2 - 4\,051 \\ &= (2\,026 + 2\,025)(2\,026 - 2\,025) - 4\,051 = 0. \end{aligned}$$

RIGHT: matome, check and exit ticket

Matome:

1. Rewriting must preserve exact equality.
2. I choose the tool that makes the remaining calculations simpler.
3. A round number is a clue, not a rule: identities and the distributive property are tools, not chapter reflexes.

Check:

$(x + 10)^2 = x^2 + 100$ is not an identity: $20x$ is missing; $x = 1$ suffices to refute it.
 $47 \times 53 \neq 2\,500$: $(50 - 3)(50 + 3) = 2\,500 - 9 = 2\,491$.

Exit ticket: $49 \times 51 = 50^2 - 1^2 = 2\,499$.

Board rule: Kadai, P1–P4, cost comparison, checks and matome remain visible until the final photograph. Only auxiliary arithmetic trials may be erased after verbalisation.

2. Storyboard: write, show, connect, keep, erase

Consistent colour code: main writing in base colour; relationship, cost criterion or invariant in an accent colour; refuted proposal crossed out but legible. Outside the auxiliary strip, no selected answer is erased during the lesson.

Time	Board state	What is written	Speech and activity	Board action / status
Before entry	T0 - discreet outline	Draw three zones; reserve space for A , B , P1–P4 and the matome, without writing any solution.	Prepare the two problem cards, identical for everyone; do not prepare any mock pupil work.	RESERVE; do not pre-solve anything.
0–8	T1 - timed fluency	After five minutes, display the answers to 4.1; record $n(\geq 6/8)$ and $n(8/8)$, not a speed ranking.	Individual work, self-marking; discuss only the structures of c) and g).	KEEP the class indicator; ERASE the displayed answers after marking.
8–11	T2 - given data and Kadai	Write $A = 10,1 \times 9,9$, $B = 38 \times 998$, then the Kadai.	“For each calculation: rewrite, name the tool and state what actually remains to be calculated.”	KEEP A , B and the Kadai.
11–20	T3 - investigation	Write no method on the main board; auxiliary strip reserved for clarifying questions.	Four minutes alone, five in pairs. Circulate; note R1–Rk and the arithmetic cost of each route.	CIRCULATE; select complete answers, without fabricating pupil work.
20–33	T4 - neriage P1–P3	Put up P1, then P2 or T, then P3. Link with “correct?” and “shorter?”.	“Do both routes preserve the number? Which one reduced the work, and where can we see that?”	KEEP P1–P3; mark T if the midpoint 518 is introduced by the teacher.
33–40	T5 - transfer P4	Add the 2026 calculation and the chain yielding zero, without expanding the squares.	Two minutes alone, pair comparison, then whole-class discussion: “which part should we transform first?”	KEEP P4 and link it to the cost criterion.
40–46	T6 - targeted checks	Display only 4.5 a) and c) as statements to test; keep counter-example and structural argument.	Vote, justification, revote. “For a), refute the ‘for all’; for c), check the numerical equality without column multiplication.”	KEEP both refutations; c) may be omitted if running late.
46–52	T7 - protected matome	Build the three sentences: equality preserved, calculation simplified, final check.	Have pupils formulate before displaying the written summary; allow copying after class agreement.	KEEP P1–P4 during copying.
52–55	T8 - exit ticket	Mini-whiteboard: 49×51 ; require transformation, tool and result.	“Which midpoint makes the calculation less costly?” Expected: $50^2 - 1^2 = 2\,499$.	PHOTOGRAPH while collecting materials.

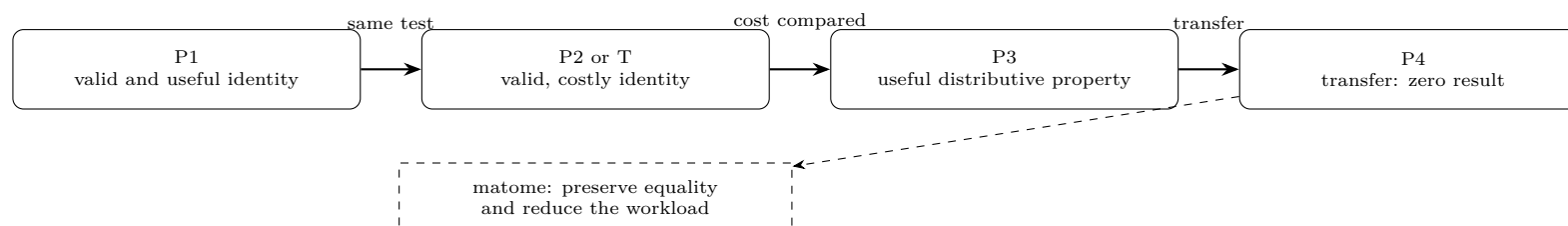
Fallback rule: display the answers to 4.1 without commenting on all of them, then omit 4.5 c). P1, P2 or T, P3, P4, the matome and the exit ticket remain non-negotiable: without the “valid / efficient” comparison, the Kadai is not resolved.

3. Managing the discussion: select, sequence, connect

ID	Anticipated output	Role in the narrative	Exact questions	Placement and status
P1	$10,1 \times 9,9 = (10 + 0,1)(10 - 0,1) = 100 - 0,01 = 99,99$.	Show a simple common midpoint: the identity genuinely reduces the cost.	"Which number is equidistant from both factors? What operations remain after rewriting?"	Left; ESS keep with midpoint 10.
P2	$38 \times 998 = (518 - 480)(518 + 480) = 518^2 - 480^2$.	Distinguish validity from efficiency: the identity is exact, but it replaces one product with two large squares.	"How did you find 518 and 480? Is the equality true? Has the calculation become shorter?"	Centre; ESS build as a class and mark T if no pupil produced it.
P3	$38 \times 998 = 38(1000 - 2) = 38000 - 76 = 37924$.	Provide an exact and genuinely economical rewrite; compare its cost with P2.	"Which round number acts on only one factor? How many easy products remain here, compared with P2?"	Centre, below P2; ESS keep and link with "more economical".
P4	$2026^2 - 2025^2 - 4051 = (2026 + 2025)(2026 - 2025) - 4051 = 0$.	Transfer the criterion without a round-number clue: the difference of squares creates an immediate product.	"Which sub-calculation should be transformed before any approximation? Why does it leave exactly zero?"	Centre right; ESS keep until the matome.

Planned selection order. P1, where identity and economy coincide → P2, exact but costly → P3, exact and economical → P4, transfer without superficial imitation. After each contribution, ask separately: "is equality preserved?" then "is the cost reduced?"

Actual answers and teacher input T. Note only R1-Rk. If the midpoint 518 does not appear, the teacher publicly asks for the average of the two factors and builds P2 with the class, under the code T. No pre-written sheet is attributed to a pupil.



Structured variation. The first two products both have a midpoint and a gap; only the cost of the resulting numbers varies. The 2026 calculation then changes its surface form while keeping the criterion "transform the part that becomes simple".

Invariant to verbalise. A transformation must preserve equality. A method is called smart only if it reduces the number of operations or replaces them with manageable calculations.

Scope of validity. "An identity is applicable" and "an identity is advantageous" are two different statements. A round number is a clue about cost, not a mathematical condition for validity.

4. Comparing the plan with the board actually built

Observed item	Planned	Actual / evidence collected
Investigation and whole-class discussion time	0–8 fluency; 8–11 Kadai; 11–20 investigation; 20–33 neriage; 33–40 transfer; 40–46 check; 46–52 matome; 52–55 exit ticket.	N/A
Outputs that emerged	P1 midpoint 10; P2 exact but costly identity; P3 economical distributive property; P4 difference of squares yielding zero.	N/A
Order of presentation	exact and useful → exact but costly → exact and useful → transfer	N/A
Useful unexpected output	note its equality, remaining cost and link with P1, P2 or P3	N/A
Question that moved thinking forward	“Is equality preserved?” then “has the workload genuinely decreased?”	N/A
Elements kept visible	A, B, Kadai, selected R1–Rk, any teacher input T, P1–P4, cost links and matome	N/A
Elements erased and when	displayed answers to 4.1 and auxiliary calculations, only after marking or verbalisation	N/A
Matome stated by pupils	A smart transformation preserves equality and makes the remaining calculations simpler; a round number is only a clue.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert photograph here
or state its filename

Photograph checklist

- ☐ The Kadai and the two compared calculations are identifiable.
- ☐ The actually selected answers R1–Rk are identifiable.
- ☐ Any teacher input T is not attributed to a pupil.
- ☐ Validity and efficiency are distinguished.
- ☐ The 2026 transfer and the checks are visible.
- ☐ The matome is understandable without the teacher’s spoken commentary.

Class indicator	$n(\geq 6/8)$ in five minutes: _____	Deferred copying	Actual start time: _____
Local decision	Element to keep: _____	Single adjustment	Element to modify: _____

Bansho keikaku - planned board

1. Before the lesson: intention and planned final board

Type: problem-based lesson through variation of the same structure

Kadai	How can we transform each of the three sums into a product without losing a term or a sign? Which repeated block is common to all three, and how can we prove the equality for every value of x ?	Objective	Take out a common factor, distinguish between a valid factorisation and a further extraction, then justify the identity by re-expanding before opening the route of notable identities.
Equipment	Course notes part 4; exercise sheet 3.1 b), 3.2 b) and d) and 2.3 a) first item; mini-whiteboards.	Outputs	P1 valid product but partial extraction; P2 further extraction; P3 common bracket and sign; P4 factor revealed by writing a square as a product.

ESS essential function **CONT** contingent form **DECL** teacher input only if needed. P1–P4 denote anticipated functions; on the actual board, only observed pieces of pupil work are noted C1, C2, etc. Any contribution added by the teacher is signed T, never attributed to a pupil.

Planned final board at the end of the lesson.

IDEAL BOARD (final state): to project or reproduce

LEFT: shared problem

$3x(4x - 5) = 12x^2 - 15x \leftarrow$ factorise

Kadai: How can we transform each of the three sums into a product without losing a term or a sign? Which repeated block is common to all three, and how can we prove the equality for every value of x ?

$$F_1 = 12x^2 - 15x$$

$$F_2 = (x + 5)(4x - 2) - (x + 5)(9x + 1)$$

$$F_3 = (4x + 5)^2 - 5(4x + 5)$$

CENTRE: compared outputs

P1: $F_1 = 3(4x^2 - 5x)$: valid product; x remains common.

P2: $F_1 = 3x(4x - 5)$: further extraction.

$$\begin{aligned} \text{P3: } F_2 &= (x + 5)[(4x - 2) - (9x + 1)] \\ &= (x + 5)(-5x - 3). \end{aligned}$$

$$\begin{aligned} \text{P4: } F_3 &= (4x + 5)[(4x + 5) - 5] \\ &= 4x(4x + 5). \end{aligned}$$

RIGHT: proof and second route

$kA + kB = k(A + B)$ and $kA - kB = k(A - B)$, by the distributive property. Re-expanding proves the equality for all x .

Test value: mismatch $\Rightarrow$ false; match $\nRightarrow$ proof.

Before identities: common factor?

Otherwise: terms; squares; double product.

$$x^2 + 10x + 25 = (x + 5)^2$$

$$x^2 - 8100 = (x + 90)(x - 90)$$

Board rule: keep P1 and P2 simultaneously: the goal is to compare two true expressions, not to turn P1 into a mistake. The problem, the selected pupil work, the distributive property chain and the matome remain visible until the final photograph.

2. Storyboard: write, show, connect, keep, erase

Consistent colour code: main writing in base colour; relationship, logical connector or invariant in an accent colour; error crossed out but legible. Outside the auxiliary strip, no selected output is erased during the lesson.

Time	Board state	What is written	Speech and activity	Board action / status
Before entry	T0 - layout	Reserve F_1, F_2, F_3 on the left, four pieces of work in the centre, proof and second route on the right.	Prepare the three expressions without any visual cue on the factor.	RESERVE; do not circle the block to be discovered.
0–6	T1 - reversed arrow	Expand $3x(4x - 5)$, then reverse the arrow towards $12x^2 - 15x$.	“What changes when going the other way?”	KEEP the equality that sets the proof requirement.
6–10	T2 - Kadai	Write F_1, F_2, F_3 and the central question.	Silent reading; no method given.	KEEP all three expressions intact on the left.
10–21	T3 - investigation	Nothing in the solution zones; record only the time and the numbers of the spotted pieces of work.	5 min alone, 6 min in pairs; product, circled block, justification.	OBSERVE; select C1–C4 without preparing fictional pupil work.
21–34	T4 - neriage	Put up P1 then P2 on F_1 , P3 on F_2 , P4 on F_3 ; link each factor to its two occurrences.	Compare “already a product” and “further extraction”; address the sign issue only if it arises.	KEEP P1–P4; actual error crossed out but legible.
34–41	T5 - proof	Write $kA + kB = k(A + B)$ and $kA - kB = k(A - B)$, then re-expand one output. Write match $\Rightarrow$ proof.	“Which line holds for all x ? What does a test value allow us to do?”	KEEP the general chain; erase only an auxiliary numerical calculation.
41–48	T6 - second route T	Write “common factor first”, then the three questions and the two contrasting identities.	Have pupils work out $x^2 + 10x + 25$, then $x^2 - 8100$.	SIGN T if the input comes from the teacher; do not rename it P.
48–53	T7 - copying	Dictate the short written summary after it is formulated by the class.	Silent copying; monitor equality signs and logical negation.	PROTECT the matome and defer any homework announcement.
53–55	T8 - exit ticket	Write up $7x - 35$ and $x^2 - 36$.	Name two routes, factorise, mentally re-expand one answer.	FINAL PHOTO after collecting mini-whiteboards.

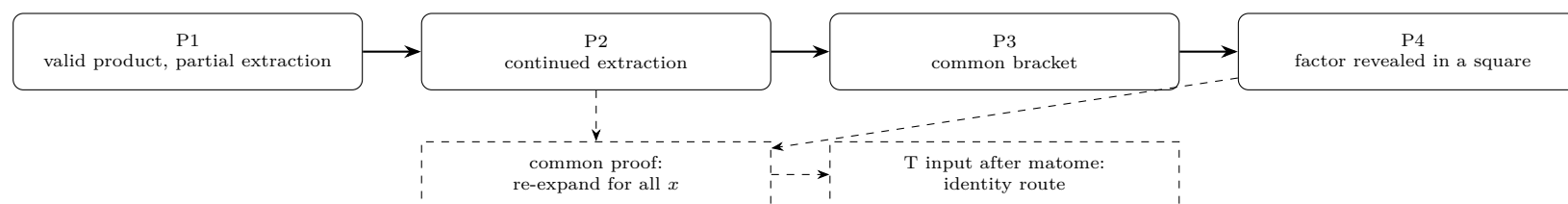
Fallback rule: reduce the second route to the single perfect square $x^2 + 10x + 25$. Do not cut the investigation, the P1–P4 comparison, the general re-expansion or the distinction between a spot check and a proof.

3. Managing the discussion: select, sequence, connect

ID	Anticipated output	Role in the narrative	Exact questions	Placement and status
P1	$F_1 = 3(4x^2 - 5x).$	Help pupils recognise a true factorisation that still leaves a common factor visible.	“Is the main operation an addition or a multiplication? Does this expression already meet the definition?”	Centre, first answer; keep without any error mark.
P2	$F_1 = 3x(4x - 5).$	Compare two degrees of completion and bring out further extraction.	“What is still common inside the brackets of P1? How does P2 rebuild each term?”	Below P1; link the remaining x then keep both.
P3	$F_2 = (x + 5)[(4x - 2) - (9x + 1)] = (x + 5)(-5x - 3).$	Extend the common factor to an entire bracketed expression and work on the scope of the minus sign.	“What must $(x + 5)$ be multiplied by to recover the second product? What signs do we get when simplifying?”	Centre; keep. Only cross out a sign error if an actual piece of pupil work contains it.
P4	$F_3 = (4x + 5)[(4x + 5) - 5] = 4x(4x + 5).$	Reveal a factor hidden by the square notation without changing the invariant.	“What product can replace the square? Which block then appears in both terms?”	Upper centre; box both occurrences of $(4x + 5)$ and keep.

Planned selection order. Valid but partial product $\rightarrow$ continued extraction $\rightarrow$ bracket factor $\rightarrow$ factor made visible by rewriting. This order keeps the same problem and only increases the camouflage.

Essential / contingent. Functions P1–P4 and their connection are necessary; the authors, the syntax and the appearance of a sign error are contingent. If P1 does not appear, the teacher may write T: “could we stop at $3(4x^2 - 5x)$?”, without inventing a pupil’s work.



Organised variation. V1 leaves a letter factor inside the bracket; V2 extracts it; V3 replaces the monomial factor with a bracket; V4 hides the second occurrence inside a square.

Invariant to verbalise. In $kA + kB = k(A + B)$ and $kA - kB = k(A - B)$, k multiplies each initial term and the bracket contains exactly what is needed to reconstruct it. Re-expanding establishes the equality for all admissible values.

Scope of use. A factor can only be taken out if it multiplies all terms. A test value can refute an identity through disagreement; its agreement at a single point cannot prove it.

4. Comparing the plan and the board actually built

Observed point	Planned	Actual / evidence collected
Search and whole-class discussion time	0–6 arrow; 6–10 Kadai; 10–21 search; 21–34 neriage; 34–41 proof; 41–48 second route; 48–53 copying; 53–55 exit.	N/A
Productions observed	P1 valid but partial product; P2 continued extraction; P3 bracket and sign; P4 square rewritten as a product.	N/A
Order of presentation	valid partial → more complete → compound factor → hidden factor → general proof	N/A
Useful unexpected production	strategy, equivalent expression or objection to link to the Kadai	N/A
Question that moved thinking forward	“Is this already a product?”; “Which line proves equality for all x ?”; “What does a test value actually allow us to do?”	N/A
Elements kept visible	F_1, F_2, F_3 , C1–C4, simultaneous P1/P2, distributive property, scope of testing, matome	N/A
Elements erased and when	single numerical calculation only, after writing that it does not constitute a proof	N/A
Matome formulated by pupils	Factorising means writing an equal product; the factor must multiply all terms and re-expanding establishes general equality.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert the photograph here
or state its file name

Photograph checklist

- ☐ The problem and the goal are identifiable.
- ☐ C1–C4 are distinguishable and linked to functions P.
- ☐ A refuted error is not confused with the result.
- ☐ Single-value agreement and general proof are distinguished.
- ☐ The invariant or condition is visible.
- ☐ The matome is understandable without the teacher's spoken commentary.

Class indicator

Observed production or change of mind: _____

Deferred copying

Actual start time: _____

Local decision

Element to keep: _____

Single adjustment

Element to modify: _____

Bansho keikaku - planned board

Chapter 01B - S7 - Algebraic expressions and notable identities

1. Before the lesson: intention and planned final board

Type: classification by variation, comparison of routes and logical transfer

Kadai	What exact verdict can we justify for each of the four cards, and on what clues? When several routes exist, how can we compare them?	Objective	Classify four boundary cases, delimit each verdict and distinguish a substitution that refutes from a re-expansion that proves.
Equipment	Cards A–D from 3.3; exercise sheet 3.6; photographer's poster; mini-whiteboards; clues/verdict/reason grid.	Productions	P1 reversed order; P2 two exact routes; P3 provisional limit; P4 chapter identity not applicable. Louise is then a transfer, not a fifth answer to the classification.
ESS essential function CONT contingent form DECL teacher input. P1–P4 denote anticipated mathematical functions, not guaranteed pupil work. While circulating, code actual responses R1–Rk; if an essential input is missing, write it under the label T, without attributing it to a pupil.			

Planned final board at the end of the slot.

IDEAL BOARD (final state): to project or reproduce

LEFT: Kadai and cards

Kadai: What exact verdict can we justify for each of the four cards, and on what clues? When several routes exist, how can we compare them?

A $64 - 80x + 25x^2$

B $16x^2 - 144$

C $x^2 - 13$

D $x^2 + 81$

Clues: number of terms; squares; sign; double product; common factor.

CENTRE: P1–P4 compared

P1 $A = (8 - 5x)^2$: order has no effect on the structure.

P2 $B = (4x + 12)(4x - 12)$
 $= 16(x + 3)(x - 3)$: two exact products; the second path is more direct.

P3 C: not yet with the available numbers.

T Ch. 02: $13 = (\sqrt{13})^2$.

P4 D: none of the three identities in the chapter applies; leave unfactorised here.

RIGHT: transfer and matome

Louise: $E = 4x^2 - 4x + 1$ and $4x(x - 1)$.
 $x = 1$: $1 \neq 0 \Rightarrow$ claim refuted.
 $(2x - 1)^2 = 4x^2 - 4x + 1$ for all x .

Matome: disagreement at one value $\Rightarrow$ false; agreement at one value $\neq$ proof; re-expanding establishes the identity.

Material rule: Kadai, cards, selected productions, reasons, T input and matome remain visible until the final photograph. Only the auxiliary strip of unselected calculations may be erased after verbalisation.

2. Storyboard: write, show, connect, keep, erase

Consistent code: main writing in base colour; relationship, condition or logical status in an accent colour; error crossed out but legible. Pupils search before answers are written up.

Time	Board state	What is written	Speech and activity	Material action / fate
Before entry	T0 - layout	Reserve the three zones; prepare the four face-down cards and an auxiliary strip.	No result or production P is pre-written.	RESERVE; do not reveal the verdicts.
0–6	T1 - back-and-forth	Two lines from 3.5; under each, “valid for all x ? why?”.	Name the direction of the arrow; distinguish single-value agreement from proven equality.	ERASE the review calculations; KEEP “agreement $\nRightarrow$ proof”.
6–9	T2 - devolution	Turn over A–D; write the Kadaï and the columns “clues / verdict / reason”.	Silent reading, then rephrasing without any answer from the teacher.	KEEP Kadaï and cards until the end.
9–20	T3 - search	Nothing in the central zone; only note, on the circulation plan, R1–Rk and possible links.	5 min individual, 6 min pairs. Prompt: “with what tools? what can you actually justify?”	DO NOT CORRECT; select actual productions.
20–34	T4 - neriage	Place P1–P4 in the order A, B, C, D; link each verdict to its clues. Add $T : 13 = (\sqrt{13})^2$ only after P3.	Compare the four responses to the same problem; have both routes for B validated.	KEEP P1–P4; explicitly mark T.
34–40	T5 - matome	Box the differences: order/structure; accuracy/economy; provisional limit/non-applicability within the chapter.	Have pupils formulate before writing.	PROTECT the matome; no copying yet.
40–49	T6 - Louise	Write E , Louise’s claim, the disagreement at $x = 1$, then the re-expansion of $(2x - 1)^2$.	Vote; refute; correct; ask what each check allows us to conclude.	CROSS OUT without erasing; link “test” to refutation and “re-expansion” to proof.
49–53	T7 - summary	Stabilise the two sentences on scope and logical status.	Deferred copying, full board visible.	PHOTO after copying the written summary.
53–55	T8 - exit	In the auxiliary strip: $25x^2 - 81$ or $x^2 + 16$.	Full individual verdict and reason.	COLLECT; the exit task may then be erased.

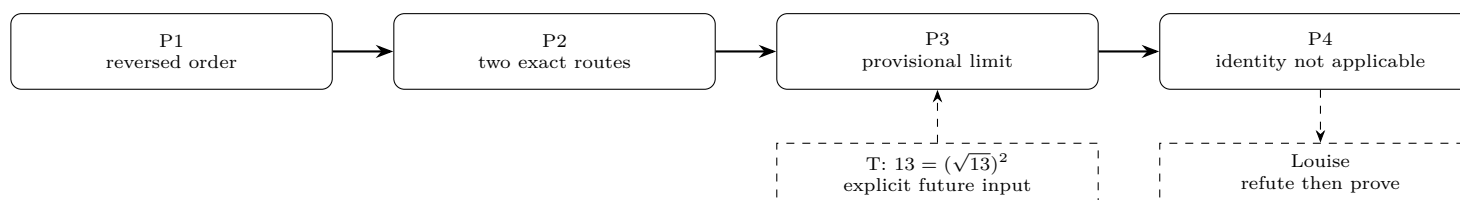
Time consistency check: if time runs short, reduce the review to 3 min and Louise to 7 min. Do not cut the shared search, P1–P4, the matome or the refute/prove distinction. 3.4 is not a second lesson to be squeezed into this slot.

3. Generating the discussion: select, order, connect

ID	Anticipated production	Role in the narrative	Exact questions	Placement and fate
P1	A: $64 - 80x + 25x^2 = 25x^2 - 80x + 64 = (5x - 8)^2 = (8 - 5x)^2$.	Recognise a perfect square despite the reversed order.	“What are the two squares? Is the middle term equal to $2 \times 8 \times 5x$? Does the order change the sum?”	Top centre; keep 8^2 , $(5x)^2$ and the double-product check.
P2	B: $(4x + 12)(4x - 12)$ and $16(x + 3)(x - 3)$.	Compare two exact factorisations without turning the first into an error.	“How can we prove that both products give B back? Which one makes the factor 16 visible earliest?”	Below P1; place the routes side by side and link with “exact; different costs”.
P3	C: “I cannot yet write 13 as a square with the available numbers”.	Delimit a provisional limit of the tools.	“Is the word never justified? What new numerical object could lift the block?”	Bottom centre; keep the pupil phrasing. Then add $T : 13 = (\sqrt{13})^2$, never under an R code.
P4	D: two added terms; none of the three identities in the chapter matches; leave unfactorised here.	Distinguish current non-applicability from an absolute claim of irreducibility.	“Which exact identity are you using? What can you state with the tools of the chapter, without claiming more?”	Below P3; keep the full verdict and its scope.

Actual selection. While circulating, spot responses that fulfil the functions above. A partial or erroneous production may be selected if it makes the comparison fruitful; its author is not exposed. If P3 does not appear, reconstruct the limit collectively. The expression with $\sqrt{13}$ remains a T input.

Neriage order. P1 (structure visible despite order) → P2 (two exact routes) → P3 (provisional limit) → P4 (no current identity), then Louise as a logical transfer.



Organised variation. V1 changes the order; V2 makes two routes coexist; V3 provisionally limits the available numbers; V4 changes the sign that conditions the identity. The question and the demand for justification remain invariant.

Invariant to verbalise. A verdict concerns the exact structure and must specify its scope. Re-expanding establishes an identity for all x . A substitution that gives a disagreement refutes it; a substitution that agrees does not prove it.

Scope of use. Do not have Year 10 pupils prove that $x^2 + 81$ has no factorisation into first-degree real factors: pupils do not yet have the necessary tools. The required verdict is only “none of the three identities in the chapter applies”.

4. Comparing the plan and the board actually built

Observed point	Planned	Actual / evidence collected
Actual time	0–6 review; 6–9 devolution; 9–20 search; 20–34 neriage; 34–40 matome; 40–49 Louise; 49–53 copying; 53–55 exit.	N/A
Productions observed	P1 reversed order; P2 two routes; P3 provisional limit; P4 identity not applicable.	N/A
Actual codes and order	R1–Rk noted while circulating; R/P matches filled in after selection.	N/A
Scope of verdicts	“not yet with the available numbers” distinct from “no identity in the chapter applies”.	N/A
Comparison of P2	Both products are recognised as exact; efficiency is discussed without invalidating a route.	N/A
Teacher input	$13 = (\sqrt{13})^2$ written under T only after production P3.	N/A
Status of testing	Louise refuted by a disagreement; correct factorisation proved by re-expanding.	N/A
Elements kept / erased	Kadai, A–D, R/P, reasons, T and matome kept; auxiliary calculations alone erased.	N/A
Question that moved thinking forward	“Do the two blocks have the same status?” or another actual question to note.	N/A
Gap to address	A single priority adjustment, based on an observed production or change of mind.	N/A

Photograph of the actual board

Insert the photograph here
or state its file name

Photograph checklist

- ☐ The Kadai and the four cards are identifiable.
- ☐ The actual responses R are linked to functions P1–P4.
- ☐ Both routes for B remain visible and exact.
- ☐ The input on $\sqrt{13}$ bears the label T.
- ☐ No unjustified absolute verdict is written for D.
- ☐ The refuting test and the general proof are distinguished.
- ☐ The matome is understandable without the teacher’s spoken commentary.

Class indicator

Pupils having delimited C and D: _____

Deferred copying

Actual start time: _____

Change of mind

Pupil having distinguished test and proof: _____

Single adjustment

Element to modify: _____

Bansho keikaku - planned board

1. Before the lesson: intention and planned final board

Type: conjecture, two compared proofs and counterexample

Kadai	The sum of three consecutive integers always seems to be three times the middle number. How can we prove this for all integers without being able to test them all?	Objective	Distinguish checked cases, conjecture and proof; compare two choices of letter; use a counterexample against a universal variant.
Equipment	Course notes part 5; exercise sheet 5.3 c) and 5.4; prepared spreadsheet from -100 to 99 ; mini-whiteboards; exit slip.	Productions	P1 data and conjecture; P2 possible unwarranted conclusion; P3 proof from the first integer; P4 proof from the middle number.
ESS essential function CONT contingent form DECL explicit teacher input. While circulating, code actual responses R1–Rk. If the unwarranted conclusion does not appear, submit it as a T claim to be judged; never attribute it to fictitious pupil work.			

Planned final board at the end of the slot.

IDEAL BOARD (final state): to project or reproduce

LEFT: data and scope

Kadai: The sum of three consecutive integers always seems to be three times the middle number. How can we prove this for all integers without being able to test them all?

$$(-8) + (-7) + (-6) = -21 = 3 \times (-7)$$

$$7 + 8 + 9 = 24 = 3 \times 8$$

$$199 + 200 + 201 = 600 = 3 \times 200$$

Spreadsheet: B2=A2+1, C2=A2+2,
D2=A2+B2+C2.

200 cases are checked; the statement for all integers remains to be proved.

CENTRE: two connected proofs

R? or T: “the 200 cases are enough”: no, they do not cover the infinite set of integers.

P3, first integer n :

$$n + (n + 1) + (n + 2) = 3n + 3 = 3(n + 1).$$

P4, middle m :

$$(m - 1) + m + (m + 1) = 3m.$$

In both proofs: $m = n + 1$.

The quotient by 3 is the middle number.

RIGHT: matome and variation

Matome:

- checked cases suggest a conjecture;
- we choose any integer and state what it represents;
- a justified chain establishes the general case;
- we conclude with “for every integer n ”.

Four consecutive integers:

$1 + 2 + 3 + 4 = 10$ refutes “always a multiple of 4”.

$n + (n + 1) + (n + 2) + (n + 3) = 4(n + 1) + 2$: the remainder on division by 4 is always 2.

Material rule: simultaneously keep the data, the scope of the spreadsheet, both proofs, their relationship $m = n + 1$ and the matome. Reviewing 5.3 stays in an auxiliary strip and may be erased after verbalisation.

2. Storyboard: write, show, connect, keep, erase

Consistent code: main writing in base colour; scope, variable relationship and logical connector in an accent colour; refuted claim crossed out but legible. No pupil response is prepared in advance.

Time	Board state	What is written	Speech and activity	Material action / fate
Before entry	T0 - layout	Write the Kadai on the left; reserve data, n/m proofs, matome and variation.	Prepare the spreadsheet, the circulation grid and empty zones; no model answer.	RESERVE; do not display the proofs.
0–6	T1 - debt from S2	In the auxiliary strip: $P = (x + 1)^2 - x^2 = (2x + 1) \times 1 = 2x + 1$.	Compare with the expansion from S2: “which route best shows sum and gap?”	VALIDATE then ERASE only if space is needed.
6–9	T2 - devolution	Write the verbal conjecture and the Kadai, with no letter or general formula.	Have pupils rephrase “three consecutive integers” and “three times the middle”.	KEEP the Kadai until the photo.
9–15	T3 - data	Write up the three contrasting cases and the spreadsheet formulas; distinguish “checked” from “proved for all”.	Individual calculation, then careful wording of the conjecture.	KEEP P1; R? or T only if useful.
15–22	T4 - search	Main board unchanged; note on the plan the choices of letter and responses R1–Rk.	Alone then in pairs: choose a letter for one of the three numbers and write a proof.	ADD NOTHING that steers towards n or m .
22–36	T5 - neriage	Set up P3 then P4; link the variables via $m = n + 1$; keep a conclusion by sum of multiples if it appears.	Compare validity, length and information made visible by each proof.	KEEP both routes; mark T for any missing input.
36–43	T6 - matome and copying	Box cases / conjecture / any integer / calculation / universal conclusion.	Have pupils formulate the four steps before deferred copying.	PROTECT for seven minutes; full board visible.
43–51	T7 - variation	Write the claim about four consecutive integers, the counter-example and then $4(n + 1) + 2$.	Refute first; then establish the more precise statement about the remainder.	CROSS OUT the false universal claim without erasing it.
51–55	T8 - exit	Auxiliary strip: five consecutive integers centred on m .	Prove in one line that their sum is $5m$; collect.	PHOTO before erasing the exit task.

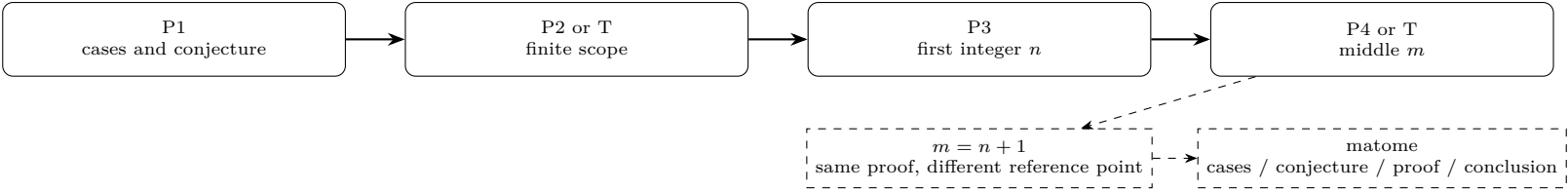
Time consistency check: if time runs short, reduce the 5.3 review to three minutes and the variation to five. Do not cut the search, the comparison of the two choices of letter, the matome or the exit task. Arthur and problem posing are not squeezed into this lesson; 5.1 is prepared for the opening of S9.

3. Generating the discussion: select, order, connect

ID	Anticipated production	Role in the narrative	Exact questions	Placement and fate
P1	The three cases, then “the sum seems to be three times the middle number”.	Bring out a precise conjecture, including for a negative starting value.	“What is the quotient by 3? Which word must we use as long as not all integers are covered?”	Left; keep as data, never as a general proof.
P2	“The 200 rows of the spreadsheet prove the property for all integers.”	Delimit what a finite check proves on an infinite set.	“Which integers have actually been checked? Does the set of integers have a last row?”	Only if observed; otherwise T claim to be judged, with no fictitious author.
P3	$n + (n + 1) + (n + 2) = 3n + 3 = 3(n + 1)$, with a quantified conclusion.	Prove from the first integer. At $3n + 3$, also invoking closure under addition of multiples of 3 gives a complete proof.	“What does n represent? Why is $n + 1$ an integer? Why is $3(n + 1)$ a multiple of 3?”	Centre; keep the chain and its logical connectors.
P4	$(m - 1) + m + (m + 1) = 3m$, with m the middle number.	Give a second proof that makes the quotient appear immediately.	“What does m represent? What is the link with n ? Which proof shows three times the middle most directly?”	To the right of P3; build under T after a whole-class prompt if missing.

Actual selection. While circulating, spot one proof that names the first integer and one that names the middle number. If only one representation appears, ask the class: “what if the letter denoted another number in the trio?”; any necessary reconstruction is marked T. An error on constants is only selected if it actually occurs.

Neriage order. P1 (conjecture) → P2 or T (finite scope) → P3 (proof from the first) → P4 or T (proof from the middle) → connection $m = n + 1$.



Organised variation. V1 negative, positive and large cases; V2 finite scope of the spreadsheet; V3 choice of the first integer; V4 choice of the middle number. The invariant is the sum equal to three times the middle.

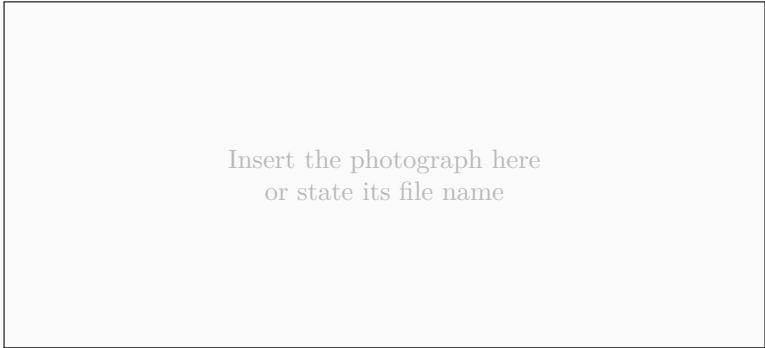
Status of $3n + 3$. This form already provides a proof if the pupil states that $3n$ and 3 are multiples of 3, and then that their sum is again a multiple of 3. Factorising as $3(n + 1)$ is a second, more compact justification, not the only way to conclude.

Logical boundary. Finitely many examples do not prove a statement about all integers, which form an infinite set. A single counterexample, however, refutes such a universal statement.

4. Comparing the plan and the board actually built

Observed point	Planned	Actual / evidence collected
Actual time	0–6 review; 6–9 Kadai; 9–15 data; 15–22 search; 22–36 neriage; 36–43 matome; 43–51 variation; 51–55 exit.	N/A
Productions observed	P1 conjecture; P2 if any; P3 proof from the first; P4 proof from the middle.	N/A
Actual codes and order	R1–Rk noted while circulating; R/P matches filled in after selection.	N/A
Scope of the spreadsheet	Cases actually checked distinguished from the statement about all integers.	N/A
Comparison of proofs	Chosen representation, validity of connectors, length and “three times the middle” information.	N/A
Teacher input	Any missing claim or representation is marked T; no pre-prepared pupil work.	N/A
Elements kept / erased	Kadai, data, R/P, both proofs, link $m = n + 1$, variation and matome kept; auxiliary calculations alone erased.	N/A
Question that moved thinking forward	“What does your letter represent?” or another actual question to note.	N/A
Gap to address	A single priority adjustment, based on an observed production or change of mind.	N/A

Photograph of the actual board



Photograph checklist

- ☐ The Kadai and the scope of the spreadsheet are legible.
- ☐ Responses R1–Rk are linked to functions P.
- ☐ The two choices of letter are not confused.
- ☐ The link $m = n + 1$ is visible.
- ☐ A T claim is not attributed to any pupil.
- ☐ The matome is understandable without the teacher’s spoken commentary.

Class indicator	Observed production or change of mind: _____	Deferred copying	Actual start time: _____
Local decision	Element to keep: _____	Single adjustment	Element to modify: _____

Bansho keikaku - planned board

1. Before the lesson: intention and planned final board

Type: homework feedback, comparing proofs and transfer under exam conditions

Kadai	Anabelle claims that the difference of the squares of two consecutive odd integers is always a multiple of 8. How can we prove this, and what does each method make visible?	Objective	Reinvest the scope of numerical trials; compare two general proofs without confusing validity, length and readability; use criteria on Brevet 5 (the brevet, French national exam at the end of Year 10).
Equipment	Two real, anonymised pupil scripts from 5.1; worked solutions for the exercise sheet in an envelope; problem 5.5 projected, exercise sheet closed during the investigation; Brevet 5; timer.	Outputs	P1 trials and conjecture; P2 expansion with conclusion via sum of multiples; P3 explicit quotient $8(n+1)$; P4 difference of two squares.
ESS essential function CONT contingent form DECL explicit teacher input. While circulating, code the actual responses R1–Rk without names or initials. Any absent route that is useful for comparison is built publicly under the label T, never attributed to a fictional script.			

Planned final board at the end of the slot.

IDEAL BOARD (final state): to project or reproduce

LEFT: return to Arthur, data and Kadai

Review 5.1:

$$5(-2x + 5) = -10x + 25;$$

$$(x - 5)^2 - x^2 = -10x + 25.$$

Trials at 2 and 3 establish two cases; the identities prove equality for every real number x .

Kadai: Anabelle claims that the difference of the squares of two consecutive odd integers is always a multiple of 8. How can we prove this, and what does each method make visible?

Let n be an integer: two consecutive odd integers are written $2n+1$ and $2n+3$.

$$5^2 - 3^2 = 16, 9^2 - 7^2 = 32, 13^2 - 11^2 = 48: \text{conjecture.}$$

CENTRE: compared proofs on 5.5

P2, expand:

$$(2n + 3)^2 - (2n + 1)^2 = 8n + 8.$$

$8n$ and 8 are multiples of 8 ; so is their sum.

P3, explicit quotient:

$$8n + 8 = 8(n + 1), \text{ with } n + 1 \text{ an integer.}$$

P4, factorise first:

$$[(2n + 3) + (2n + 1)]$$

$$\times [(2n + 3) - (2n + 1)]$$

$$= (4n + 4) \times 2 = 8(n + 1).$$

Same result; sum and gap visible.

RIGHT: matome and Brevet grid

Matome:

1. trials ground a conjecture, not the general proof;
2. divisibility can be shown via a sum of multiples or via $8 \times k$ with k an integer;
3. one method may be shorter or more illuminating without making the other wrong.

Brevet grid:

- letter and domain;
- algebraic translation;
- justified equalities;
- recognised form and integer obtained;
- quantified conclusion.

Numerical check: useful after the proof, never required to prove it.

Material rule: keep the Arthur strip, the trials, all proofs actually compared, their connectors and the matome. The brevet task remains without answers on the board during its ten minutes; its solution appears only after the signal.

2. Storyboard: write, show, connect, keep, erase

Stable code: main writing in the base colour; domain, logical connector and relationship between proofs in an accent colour. The code R1–Rk denotes anonymised pupil responses; T explicitly denotes teacher input.

Time	Board state	What is written	Speech and activity	Physical action / fate
Before entry	T0 - layout	Reserve a top strip “review 5.1”, then three zones: data/Kadai, 5.5 proofs, matome/grid.	Select two actual pupil scripts from 5.1; project only statement 5.5.	EXERCISE SHEET CLOSED; no answer or method displayed.
0–10	T1 - homework reviewed	Compare the scripts from 5.1; write both simplified expressions and the exact scope of the trials.	“What do the trials prove? Which chain holds for any real number?”	KEEP the logical conclusion; hand out the worked solutions only afterwards.
10–13	T2 - handover	Write the three tests and the Kadai, without imposing any identity or letter.	Rephrase “consecutive odd integers” and distinguish conjecture from proof.	KEEP the data; do not open the hint on the exercise sheet.
13–21	T3 - investigation	Main board unchanged; on the seating plan, code R1–Rk and the methods spotted.	Four minutes alone, four minutes in threes: write out a general proof.	DO NOT GUIDE before completing a full circuit.
21–34	T4 - neriage	Put up the routes actually present; connect $8n + 8$, $8(n + 1)$ and the difference of two squares.	“Does $8n + 8$ already allow us to conclude? What more does factoring show?”	MARK with T any reconstructed route; keep the connectors.
34–39	T5 - matome	Box the scope, two divisibility criteria and the comparison of methods.	Elicit phrasing before copying; do not require a numerical check.	PROTECT five minutes; full board visible.
39–49	T6 - Brevet 5	No answer on the board; timer visible, individual work without calculator.	Write out a), b) and c) under exam conditions.	FORBID opening the worked solutions until the signal.
49–55	T7 - return to grid	Write only $4(n + 1)$ and the counterexample $1 + 5 = 6$ after the signal.	Self-marking against the distributed worked solutions; circle one criterion mastered and one to revisit.	FINAL PHOTO then collect a sample of scripts.

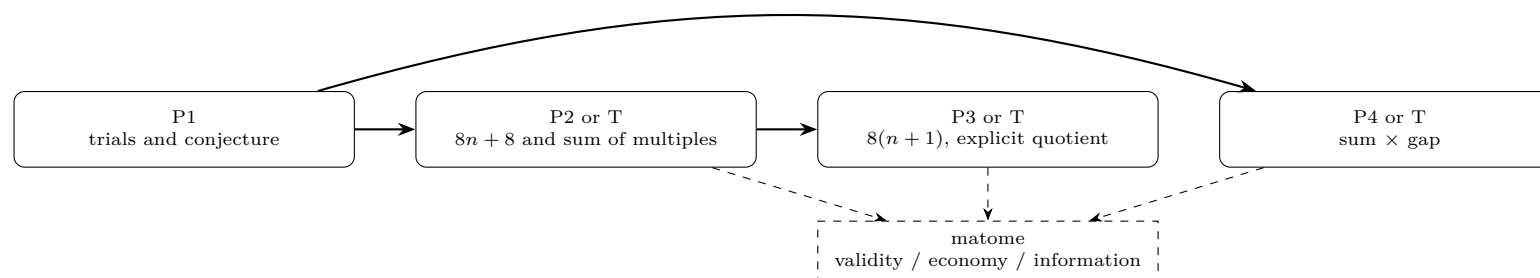
Time-consistency check: Arthur, the investigation, at least one general proof discussed, the matome and the ten minutes on the brevet task are non-negotiable. If necessary, limit the neriage to two routes and reduce the final self-marking to four minutes. 5.6 remains an out-of-slot H reserve.

3. Leading the discussion: select, sequence, connect

ID	Anticipated output	Role in the narrative	Exact questions	Placement and fate
P1	Trials give for instance 16, 32 and 48, all multiples of 8.	Prompt the conjecture without confusing it with a proof.	"What do these results suggest? Which values of n have we not tested?"	Left, under the Kadai; keep as examples only.
P2	Expanding leads to $8n + 8$, then the pupil states that $8n$ and 8 are multiples of 8 and so is their sum.	Recognise a complete proof by closure of multiples under addition.	"Is $8n+8$ merely correct, or does it already allow us to conclude? Which property of addition are you using?"	Centre left; keep as a valid proof, even without factorising.
P3	$8n + 8 = 8(n + 1)$; since n is an integer, $n + 1$ is too.	Provide a second conclusion from the same expansion, with an explicit quotient.	"Which integer plays the role of the quotient? Does this line add validity or mainly readability?"	Under P2; connect with "same proof, more compact form".
P4	$[(2n + 3) + (2n + 1)] \times [(2n + 3) - (2n + 1)] = (4n + 4) \times 2 = 8(n + 1)$.	Compare a shorter route that also displays the sum and the gap of the odd numbers.	"Which expressions play a and b ? Why is the gap equal to 2? Where can we read the sum?"	Centre right; connect to P2/P3 with "same result, different information".

Actual selection. Spot codes R1–Rk while circulating, without names or initials. Start with an output the class can understand, then connect the responses to the mathematical goal. An error goes on the board only if it was actually observed and discussing it is useful.

Essential / contingent. P1, at least one general proof and the matome are essential. P2, P3 and P4 are contingent comparison functions: if a route is missing, build it after a public prompt and mark it T.



Structured variation. V1 trials; V2 expansion and closure property; V3 same result with explicit quotient; V4 difference of two squares. Brevet 5 then varies the problem while keeping the chain: letter, calculation, recognised form, conclusion.

Invariant to elicit. To prove that an integer is divisible by d , we can write it as $d \times k$ with k an integer; we can also use previously established properties, for example the sum of two multiples of d is still a multiple of d .

Logical boundary. A numerical check can detect a discrepancy after the proof; it is neither a compulsory step of the proof nor a criterion to which a validity mark should be awarded.

4. Comparing the plan and the board actually built

Observed item	Planned	Actual / evidence collected
Actual time	0–10 Arthur; 10–13 Kadai; 13–21 investigation; 21–34 neriage; 34–39 matome; 39–49 Brevet; 49–55 review.	N/A
Outputs observed	P1 trials; P2 possible proof by sum; P3 possible factorisation; P4 possible difference of two squares.	N/A
Actual codes and order	R1–Rk noted while circulating; any absent construction marked T.	N/A
Validity of $8n + 8$	Accepted if closure of multiples under addition is stated explicitly; not reduced to “hidden factor”.	N/A
Comparison of proofs	Validity, length and information made visible are distinguished.	N/A
Question that moved things forward	“Does $8n + 8$ already allow us to conclude?” or another actual question to note.	N/A
Items kept / erased	Compact Arthur, Kadai, R/P, proofs, links, matome and grid kept; auxiliary trials alone erased.	N/A
Brevet trace	Mastered criterion and criterion to revisit identified on an actual sample of scripts.	N/A
Gap to address	A single priority adjustment, based on an observed output or change of mind.	N/A

Photograph of the actual board

Insert photograph here
or state its filename

Photograph checklist

- ☐ The return to Arthur and the Kadai are identifiable.
- ☐ Responses R1–Rk are linked to functions P.
- ☐ Any absent input is explicitly marked T.
- ☐ $8n + 8$ is not disqualified without examining the conclusion.
- ☐ Validity, length and readability are distinguished.
- ☐ The matome makes sense without the teacher’s spoken commentary.

Class indicator	Observed output or change of mind: _____	Deferred copying	Actual start time: _____
Local decision	Item to keep: _____	Single adjustment	Item to modify: _____

Bansho keikaku - planned board

Chapter 01B - S10 - feedback, choice of form and bridge question

1. Before the lesson: intent and planned final board

Type: feedback on pupil work, comparing forms, boundaries of proof and targeted revision

Kadai	For $I = (7x - 3)^2 - 25$, is there a form that is always better, or does the choice depend on the value and the purpose?	Objective	Make genuine use of Brevet 6 (French national exam at the end of Year 10); compare three exact forms using explicit criteria; distinguish verified solutions, conjecture and proof; define identity and equation precisely.
Materials	Actual pupil work on Brevet 6; set 4.1; exercise sheet 9.1; course notes part 1; study sheet; multiple-choice quiz; timer; A/B/C slips.	Pupil work	P1 expanded form; P2 factorised form; P3 comparison for $x = 1$; P4 comparison for $x = \frac{3}{7}$. Bridge question 9.1 after the matome.
ESS essential function CONT contingent form DECL public input T. Actual pupil contributions are coded R1–Rk, without names or initials. Any missing pathway is reconstructed publicly and signed T; no pupil script is prepared in advance or fabricated.			

Planned final board at the end of the lesson.

IDEAL BOARD (final state): to project or reproduce

LEFT: Brevet 6, three exact forms

Kadai: For $I = (7x - 3)^2 - 25$, is there a form that is always better, or does the choice depend on the value and the purpose?

P1, expand:

$$I = 49x^2 - 42x - 16.$$

P2, factorise:

$$I = (7x + 2)(7x - 8).$$

Both equalities are established using the notable identities; numerical agreement alone does not prove them.

CENTRE: choice of form and bridge question

P3, $x = 1$:

$$\text{starting form} = 4^2 - 25 = -9;$$

$$\text{expanded} = 49 - 42 - 16 = -9;$$

$$\text{factorised} = 9 \times (-1) = -9.$$

Preference to be justified, with no imposed winner.

P4, $x = \frac{3}{7}$:

$$\text{starting form} = 0^2 - 25 = -25;$$

$$\text{factorised} = 5 \times (-5) = -25.$$

Bridge question 9.1:

Verified: 3 and -3 satisfy $x^2 - 9 = 0$;

Proved: $x^2 - 9 = (x + 3)(x - 3)$ for every real number x ;

To justify: why there is no other solution via this route alone.

RIGHT: logical status and targeted revision

Matome.

Identity: equality stated to hold for all values in a domain.

Equation: condition on an unknown; solving means finding all its solutions.

Conjecture to be proved in Ch. 02: if a product equals zero, at least one factor is zero.

Revision:

Active study sheet + Gammes du soir (evening drills) S10 + quiz, then a single follow-up:

A calculation / B proof / C choice of form.

Practical rule: the actual pupil work from Brevet 6, its links, the three statuses for 9.1 and the matome remain visible. The future conjecture is not erased: it stays inside a dashed box that clearly indicates its status.

2. Storyboard: write, show, connect, keep, erase

Consistent colour code: main writing in the base colour; relationship, logical connector or invariant in an accent colour; error crossed out but legible. Outside the auxiliary strip, no selected piece of work is erased during the lesson.

Time	Board state	What is written	Teacher talk and activity	Board action / fate
Before entry	T0 - layout	Write the Kadai; reserve P1–P2 on the left, P3–P4 in the centre, and the bridge question and pathways A/B/C on the right.	Prepare the classroom layout; no script or error is pre-assigned.	RESERVE spaces, not authors; prepare a dashed box “conjecture to be proved”.
0–15	T1 - Brevet 6 feedback	Write up the forms and comparisons actually available, coded R1–Rk; link each equality to the identity that establishes it.	“Which chain of steps proves the equality? What criterion underpins your choice of form?”	KEEP the initial written work and annotation; any missing input is signed T.
15–19	T2 - choice matome	Elicit the formulation: same value, different information, choice depending on value and purpose.	“Is a more convenient form any truer?”	KEEP the matome; then open the worked solutions and annotate one line.
19–26	T3 - measurement 4.1	Record only the two S5 / S10 indicators after five fixed minutes.	Self-marking for two minutes; no extra fluency work.	ERASE auxiliary calculations; KEEP both pairs of indicators.
26–35	T4 - analysis of 9.1	Write the bridge question and the three columns: verified / conjectured / proved.	Three minutes alone, six in pairs; consult the written work and worked solutions to locate what each sentence establishes and the logical gap.	KEEP the wordings actually proposed and their precise justification.
35–46	T5 - boundary and summary	Place 3, –3, the factorisation identity and the conjecture in their respective columns; define identity and equation.	“What has been established? What is missing for exhaustiveness along this route?”	KEEP the dashed box; do not state it as a theorem.
46–55	T6 - targeted revision	Display the five test categories and pathways A/B/C; each pupil writes down one specific item.	“What exact task will help you improve your main weakness?”	FINAL PHOTO; collect the slips before pupils leave.

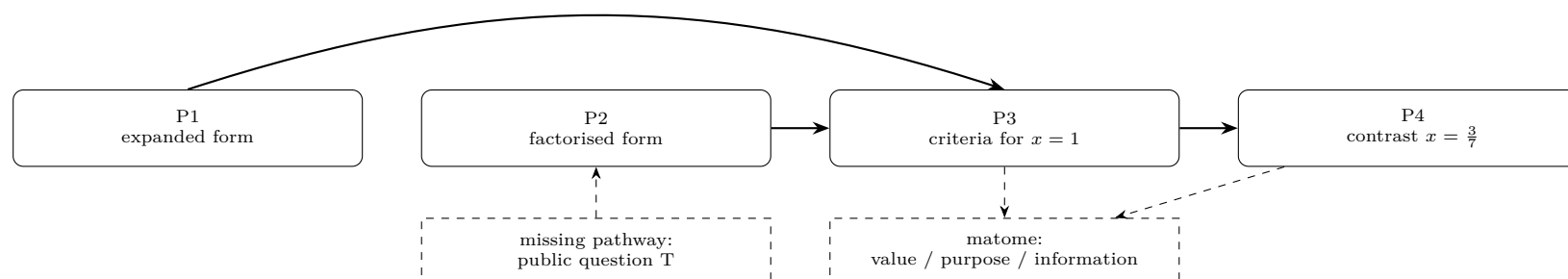
Timing consistency check: $15 + 4 + 7 + 9 + 11 + 9 = 55$. If time runs short, shorten the comparison of the 4.1 indicators; keep the Brevet 6 feedback, its matome, the proof boundary, the definitions and the revision slip. Brevet 9 and 9.2 remain outside this lesson slot.

3. Orchestrating the discussion: select, order, connect

ID	Anticipated pupil work	Role in the narrative	Exact questions	Placement and fate
P1	$I = 49x^2 - 42x - 16$, with full expansion of $(7x - 3)^2$.	Provide the sum form and establish the identity through an algebraic chain.	"Which identity are you using? Where does the -16 come from? Would agreement at $x = 0$ be enough to prove this equality?"	Top left; keep with the link to $(a - b)^2$.
P2	$I = (7x + 2)(7x - 8)$, via difference of two squares with the block $7x - 3$.	Provide the product form and compare two exact transformations.	"What are a and b ? Why the signs $+2$ and -8 ?"	Bottom left; keep with the link to $a^2 - b^2$.
P3	For $x = 1$: $4^2 - 25$, $49 - 42 - 16$ and $9 \times (-1)$ all give -9 .	Prompt discussion of a cost criterion without imposing a single preference.	"What are you counting: operations, simple numbers, the factor -1 , readability? Does your conclusion remain true for every value?"	Top centre; keep the stated criteria, not an absolute ranking.
P4	For $x = \frac{3}{7}$: starting form $0^2 - 25$; factorised form $5 \times (-5)$.	Show that two forms can be simultaneously economical and make different information visible.	"What can we see immediately in each expression? Can we still speak of 'the' best form?"	Bottom centre; link to the "value / purpose / information" matome.

Actual selection. Identify R1-Rk while marking the homework, without names or initials. An incomplete script can still serve a useful function. An error goes on the board only if it was actually observed and discussing it sheds light on the Kadai.

Essential / contingent. P1, P2, at least one value comparison and the matome are essential. P3 and P4 may come from different scripts; if a criterion is missing, the teacher asks the question publicly and signs T. No pathway is attributed to a pre-prepared script.



Structured variation. V1 sum form; V2 product form; V3 same value $x = 1$ across three expressions; V4 value $x = \frac{3}{7}$ making a zero or two opposite factors visible. What changes is the accessible information; the value of the expression remains invariant.

Transfer 9.1. Substitutions verify two solutions; the factorisation identity is proved; the general implication for a zero product remains a flagged conjecture. Another proof of exhaustiveness, provided it is complete, is not ruled out.

Definitions. Identity: equality stated to hold across an entire domain. Equation: condition on an unknown; solving means finding all its solutions. A solution set may be empty, contain one or more values (possibly infinitely many), or coincide with the entire domain.

4. Comparing the plan with the board actually built

Observed point	Planned	Actual / evidence collected
Individual work and whole-class discussion time	0–15 Brevet 6; 15–19 matome; 19–26 measurement 4.1; 26–35 analysis of 9.1; 35–46 statuses and written summary; 46–55 revision.	N/A
Pupil work that emerged	P1 expanded form; P2 factorised form; P3 criteria for $x = 1$; P4 contrast for $x = \frac{3}{7}$; wordings for 9.1 sorted by logical status.	N/A
Order of presentation	two exact forms $\rightarrow$ compared criteria $\rightarrow$ matome $\rightarrow$ transfer and proof boundary	N/A
Useful unexpected contribution	another selection criterion or another rigorous proof of exhaustiveness	N/A
Question that moved the lesson forward	“What does this form make visible?”; “What is verified, conjectured or proved?”	N/A
Elements kept visible	Kadai, R1–Rk, links to identities, criteria, three statuses, matome and pathways A/B/C	N/A
Elements erased and when	auxiliary calculations from 4.1 only, after self-marking; no selected work or labelled conjecture	N/A
Matome formulated by the pupils	A form is chosen according to value, purpose and information; an identity and an equation have different quantifications.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert photograph here
or state its filename

Photograph checklist

- ☐ The problem and the goal are identifiable.
- ☐ R1–Rk are distinguishable and linked to functions P.
- ☐ A refuted error is not confused with the result.
- ☐ Links and logical connectors are written out.
- ☐ Verified, conjectured and proved are not confused.
- ☐ The matome is understandable without the teacher's spoken commentary.

Class indicator

Observed work or change of mind: _____

Deferred copying

Actual start time: _____

Local decision

Element to keep: _____

Single adjustment

Element to modify: _____

Bansho keikaku - planned board

Chapter 01B - S11 - Algebraic expressions and notable identities

1. Before the lesson: intent and planned final board

Type: test followed by a deferred diagnostic clinic on an actual piece of pupil work

Kadai	For the deferred feedback: which actual error should we choose, where does its first breakdown occur, and what rule will prevent it from happening again?	Objective	During the test, assess unaided the structure of an expression, opposites, expansion, factorisation, proof and clever mental calculation; during the feedback session, diagnose an actual error and build a transferable prevention rule.
Materials	Checked V3 test paper and 20-mark scheme; answer sheets; timer. For the feedback session: scripts analysed by codes N/D/F/P/C, fully anonymised written extract, scripts placed face down and error logs.	Functions P1–P4	P1 faithful restatement; P2 competing diagnoses and first breakdown; P3 minimal repair with appropriate check; P4 prevention rule tested on a related case.
ESS essential function CONT contingent form DECL teacher intervention if a function is missing. P1–P4 denote functions in the discussion, not four pre-assigned scripts. Contributions actually heard are recorded as R1, R2,... without names or initials, then linked to functions P.			

Planned board for returning scripts: not to be displayed during the test.

IDEAL BOARD (final state): to project or reproduce

LEFT: written extract and diagnoses

Kadai: For the deferred feedback: which actual error should we choose, where does its first breakdown occur, and what rule will prevent it from happening again?

[actual written extract, copied faithfully and fully anonymised]

P1: “What does this written extract say exactly?”

P2: “Where is the first unjustified equality or statement?”

Main codes from the test:

N ___ D ___ F ___ P ___ C ___

CENTRE: repair and check

P3: “What is the smallest change that makes the line correct?”

“Which check is appropriate here?”

- re-expand; ○ substitute;
- check the scope of the conclusion.

Write down the check actually chosen and its outcome.

RIGHT: prevention and priority

P4: “In what related problem could this error recur?”

Prevention rule:

When opening your script:

my priority code: _____

the circled question: _____

For the next lesson: a single complete entry in the error log.

Practical rule: problem, selected contributions, refuted errors, links and matome remain visible until the final photograph. Only auxiliary calculations may be erased after validation and verbalisation.

2. Storyboard: write, show, connect, keep, erase

Consistent colour code: main writing in the base colour; relationship, logical connector or invariant in an accent colour; error crossed out but legible. Outside the auxiliary strip, no selected piece of work is erased during the lesson.

Time	Board state	What is written	Teacher talk and activity	Board action / fate
Before / 0–3	T0–T1 - neutral board, settling in	Before entry, no P zones and no feedback layout. Display only “50 min: no calculator”, the mark scheme and the handing-in rule; have pupils read through the entire paper.	“In the proof question, define your letter precisely before calculating.” No other content-related talk.	REMAINS during the test: neutral instructions and time markers only.
3–53	T2 - Individual sitting, 50 min	Show no method or result; note out of sight the items causing widespread difficulty and scripts finished very early.	No content questions; rephrase an instruction without steering the method.	ADD NOTHING that could serve as an implicit hint.
53–55	T3 - Collection	Announce the end, have pupils check their name and page count, then collect without public correction.	“Check your name and make sure all pages are handed in.”	ERASE the countdown; keep the teacher's observations out of sight.
Before feedback 0–2	T4 - prepared actual extract T5 - restatement	Copy an extract from the scripts faithfully; remove name, initials, number and any recognisable detail. Place scripts face down. Write up R1, R2, . . . : faithful restatements of what is written, without correction.	If the chosen extract is too thin, select another real script; never fabricate one. “What does this extract say exactly? Which written elements are you relying on?”	KEEP the raw extract. Scripts remain closed until the ninth minute. LINK the restatements to the corresponding line.
2–5	T6 - competing diagnoses	Keep two locations if they emerge; have the class circle the first unjustified equality or statement and assign a code.	“Where does the problem start? Can both diagnoses be right at the same place?”	CIRCLE the first breakdown only after collective justification.
5–8	T7 - repair and check	Write the minimal change and then the appropriate check: re-expanding, refuting substitution or checking logical scope.	“What minimal change is enough? Which check answers the exact doubt?”	LINK extract, repair and check outcome; keep the error legible.
8–10	T8 - prevention then priority	8–9: rule tested on a related case. 9–10: open scripts, circle one priority question and note its code.	“Does our rule work on this related case?” Then: “What is your single personal priority?”	FINAL PHOTO before opening scripts; a single error-log entry required for the next lesson.

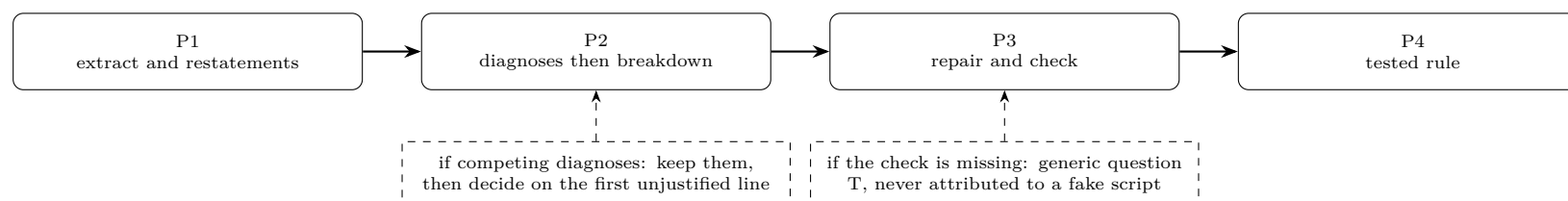
Timing consistency check: the test starts at the third minute and ends at the fifty-third: the announced 50 minutes are guaranteed in full. The deferred feedback fits into ten minutes because it deals with a single extract, a single breakdown and a single personal priority; it does not require correcting the entire script on the spot.

3. Orchestrating the discussion: select, order, connect

ID	Extract or function to obtain	Role in the narrative	Exact questions	Placement and fate
P1	The actual extract, fully anonymised, followed by one or more faithful restatements.	Separate observable writing from assumed intent and judgement.	What does this extract say exactly? Which symbols or words are you relying on?	Left; keep the extract intact and record the actual answers R1, R2,... anonymously.
P2	Two possible diagnoses or locations if the class suggests them; then the first unjustified line and its code.	Turn disagreement into an object of comparison and locate the breakdown before repairing it.	Where does the problem start? Was an earlier line already unjustified? Which code describes this breakdown?	Left to centre; keep competing diagnoses, then circle the one established by the written evidence.
P3	The minimal change and the check that addresses the specific doubt.	Repair locally without rewriting the whole script; distinguish proving an identity, looking for a counterexample and checking logical scope.	What minimal change is enough? Should we re-expand, substitute a value, or check the scope of the conclusion?	Centre; link extract, repair, chosen check and check outcome.
P4	A prevention rule formulated by the class and tested on a related example with the same structure.	Check that the rule reaches beyond the script without becoming a vague slogan.	In what related problem could the error recur? Does the rule tell us what to do there?	Right; keep the rule only after the related test, then open the scripts and choose a personal priority.

Planned selection order. Exact extract → restatements → competing diagnoses → established first breakdown → minimal repair and appropriate check → rule tested on a related case. The order concerns functions; it presupposes neither four pupils nor four scripts.

Essential / contingent. All four functions are needed for the diagnostic clinic; the chosen error, the number of actual responses and their wording remain contingent. On the board, record R1, R2,... without names or initials, then fill in afterwards how they map onto P1-P4.



Structured variation. V1 actual extract and restatements; V2 competing diagnoses then first breakdown; V3 minimal repair and check chosen according to the nature of the doubt; V4 rule tested on a related case.

Invariant to have pupils verbalise. Diagnosis focuses on an observable line of writing, not on a person or an assumed intention. The repair must address that breakdown and then be checked by a method whose scope is made explicit.

Scope of validity. The feedback board is prepared only after reading the scripts and becomes visible only after the test. Only one actual, fully anonymised extract is examined. A missing function may be prompted by a teacher question T, never simulated by an invented script.

4. Comparing the plan with the board actually built

Observed point	Planned	Actual / evidence collected
Individual work and whole-class discussion time	0-3 settling in; 3-53 individual sitting; 53-55 collection. Feedback: 0-2 restate; 2-5 diagnose; 5-8 repair and check; 8-9 test a rule; 9-10 open scripts.	N/A
Contributions that emerged	R1-Rk actually heard; post-hoc mapping onto P1 restatement, P2 diagnoses, P3 repair and check, P4 tested rule.	N/A
Order of presentation	extract → restatements → competing diagnoses → first breakdown → repair and check → tested rule	N/A
Useful unexpected contribution	small free space; note its function before deciding to keep it	N/A
Question that moved the lesson forward	During the test: no content questions. During feedback: copy the exact question that shifted the class from a diagnosis to evidence based on the written work.	N/A
Elements kept visible	actual extract, R1-Rk, first breakdown, repair, check outcome and tested rule	N/A
Elements erased and when	auxiliary trials only, after explicit choice of the check; never the initial extract or a discussed diagnosis	N/A
Matome formulated by the pupils	To correct my work, I locate the first unjustified line, make the minimal repair, and then choose a check suited to what I want to establish.	N/A
Gap to address	a single priority adjustment to note	N/A

Photograph of the actual board

Insert photograph here
or state its filename

Photograph checklist

- ☐ The problem and the goal are identifiable.
- ☐ Answers R1-Rk are distinguishable and linked to functions P afterwards.
- ☐ No name, initial or recognisable detail appears.
- ☐ A refuted error is not confused with the result.
- ☐ The first breakdown, the repair and the check are linked.
- ☐ The rule is accompanied by its related case.
- ☐ The matome is understandable without the teacher's spoken commentary.

Class indicator

Observed work or change of mind: _____

Opening scripts

Actual time: _____

Local decision

Element to keep: _____

Single adjustment

Element to modify: _____