

Chapter 01A: Lesson guide

Similar triangles (Enlargements and reductions) Year 10

Teacher document: detailed version of 2026-07-09, homework recalibrated to the Charter on 2026-07-10: evergreen (no reference to any given year's timetable)

CONTENTS

0. Timeframe: independent of the timetable	1
0 bis. What this guide owes to mathematics education research	2
0 ter. P1–P2 matrix and P9 audit of the chapter	3
1. Overview	4
2. Chapter equipment and logistics	6
3. The S0 diagnostic, item by item	7
4. Detailed lesson guides	8
S0: Year 9 reactivation and discovery (format A, 55 min)	8
S0 (variant): phase 5 “two given angles, free sizes” (alternative script)	11
S1: The Proof (format A, 55 min: key lesson of the chapter)	12
S2: Calculating lengths, the butterfly and the Brevet question (format B, 55 min)	15
S3: Configurations, SAS for similarity, and the vote (format B, 55 min)	18
S4: Enlargement and reduction, k, k^2, k^3 (format A, 55 min)	21
S5: Critical thinking and the Problem Wheel (format B, 55 min): end of the scheduled R selection	23
S6: Scratch: predicting, drawing and justifying similarity (55 min)	26
S7: Test (50 min in a 55-min slot)	28
5. Ongoing classroom routines (for display)	29
5 bis. Three teaching moves (standard kit)	30
6. Remediation script: the 6 major errors of the chapter	30
7. Required oral vocabulary (across the lessons)	30
8. Ongoing formative monitoring and assessment	31
9. Pacing and management: margins, priorities, contingencies	31
10. What this guide assumes is in place	32

This guide is designed to be *the only document open during the lesson*: each sheet contains the script, the oral questions, the problems used with their quick answers, the board plan and the word-for-word written summary. It complements the Work Plan (the what) and builds on: the roadmap of the teacher version of the exercise sheet (**pink core**, to be covered in class and corrected on the board: **1.4, 2.2, 3.5, 3.6, 4.2**; the other exercises led or assigned in this guide: 1.1, 1.2, 1.3, 1.5, 2.1, 2.3, 2.4, 2.5, 3.1–3.4, 4.1, 5.1–5.3, the brevet (French national exam at the end of Year 10) exercises 6.1–6.6: belong to the bank to choose from, not the core), the extension exercises from the sorted bank (part 7), and the A/B formats and routines of the Year 10 teaching model.

0. Timeframe: independent of the timetable

This guide describes a linear sequence of standard 55-minute periods. The statutory allocation for Year 10 is *3 h 30 per week*: the sequence must be adjusted to the actual timetable (period length, groups, bank holidays). Four periods a week are only a benchmark when the timetable provides them; lessons S0 to the final lesson run in order, without assuming a fixed weekly pattern.

Chapter budget: 8 lessons, or about 2 weeks. This is not a start-of-year chapter: S0 does not need to absorb setting up classroom routines (already established in chapter 00); it directly serves to reactivate Year 9 material and launch the topic. *The test takes up a full period* (S7: 50-min paper + changeover), counted within the 8; returning marked papers takes the first 10 minutes of the following period (opening of chapter 01B), not an extra period.

Three scheduling principles, valid for any timetable.

- If there is a double period** (two consecutive lessons): pair a new-content lesson with its practice lesson (S0+S1, S2+S3, S4+S5). If there is none, nothing changes: the pairs also work one or two days apart.
- The test* (50 min) goes in a single period, preferably the *last of the week*: papers can be marked before the next period; returning papers = the first 10 minutes of the next lesson.
- Never place S1 (the Proof) right before a break (weekend, holiday) if the consolidation in S2 cannot follow within

48 h: a proof that is not reinvested is forgotten.

Annual adjustment (the only thing to do at the start of each school year): map $S0 \rightarrow S7$ onto the actual periods while following these three principles. Everything else in the guide is stable.

0 bis. What this guide owes to mathematics education research

Research findings inform classroom practice, but they do not prove that a timed script is optimal in this class. We therefore distinguish three statuses: **[S] finding** (a result or description actually reported by research), **[T] reasoned transposition** (adaptation to Year 10 content) and **[D] local decision to evaluate** (timing, frequency, threshold or organisation). Pink rows flag a benchmark to question; they do not count as local experimental validation. In the table, the finding falls under [S]; the teaching choice and its placement fall under [T], and any numerical calibration under [D].

Transposition or decision in the guide	Finding retained	Concrete trace in this chapter
Open each practice lesson with a short drill, timed with a visible timer, corrected immediately on the board by pupils: never postponed, never collected	[S] A teaching tradition, described in a given cultural context, combines speed, accuracy and practice. [T] Here, a short classroom fluency drill is tested as reactivation; the timer is neither proof of understanding nor the sole goal. [D] Maintained, lightened or untimed depending on observed accuracy and verbalisation.	1.1 (6 min, S2), 3.1 (8 min, S4); opening fluency drills of S1–S6
Build each new-content lesson around A SINGLE substantial situation; leave the whole path (trials, mistakes, solutions) written on the board until the summary	[S] The Japanese “structured problem solving” lesson: one problem, an investigation, a whole-class discussion of methods, then “highlighting and summarizing the main point”, the matome, to which four functions are attributed; experienced teachers make elaborate use of the board. [T] “Nothing is erased before the summary” is a local rule, not a finding. [D] Nor does the finding say that the matome is never skipped: it is the final exercise or extension that is optional there, “if necessary and if time permits”.	S0 (the conjecture), S1 (the Proof), S4 (the square and k^2); “board plan” boxes
Run new-content lessons in four phases: launch (hatsumon), investigation while the teacher circulates without correcting (kikan-shidō), ordered whole-class discussion (neriage), written summary never sacrificed (matome)	[S] During the investigation, the teacher spots pupils with good ideas to call on them afterwards <i>in a chosen order</i> during the whole-class discussion; those who already have a solution are invited to look for another. [T] The order chosen here (from most naive to most expert) is a local choice: the finding does not specify it.	Sheets S0/S1/S4; prescribed speaking order (1→4) in S1 (building the proof)
Before each discovery lesson, write out the list of expected responses (at least four, including two incorrect ones [D]) and decide the speaking order in advance	[S] The same textbook page given to teachers at three levels of expertise highlights two decisive skills: conducting “structured problem solving” and <i>anticipating pupils’ responses</i> . [D] The quota “four including two incorrect ones” is a local decision: the finding sets no number and establishes no causal link between improvisation and funnelling questioning.	“Anticipation” section of each new-content sheet
Choose ONE target mistake per lesson, let it happen, display it without comment, then have the class invalidate it: the teacher never decides in the pupils’ place	[S] Funnelling (the pupil looks for the expected answer instead of thinking mathematically) is contrasted with focusing, where open questions get them to build and explain. Actual scope: all 8 observed Hong Kong lessons <i>are</i> funnelling, and only 4 out of 11 Shanghai lessons attempt focusing; in those, when a pupil gets stuck, the teacher asks another pupil the same question. [T] Displaying the mistake and having the class invalidate it is the local transposition.	S0 (pairing at random), S2 ($BC = 6,9$ in the bow-tie), S3 (pairing by letters), S5 ($56 t = k$ instead of k^3); script part 6
Work in families where ONLY ONE thing changes at a time (the configuration, the context), and close each family with “what changes? what never changes?”	Variation teaching (bianshi): what is essential is discerned only if the non-essential varies against an invariant background. Formats: one problem, several solutions; one method, several contexts.	$2.2 \rightarrow 2.3 \rightarrow 2.5$ (same method, changing configurations); the brevet 6.1/6.2 (same skeleton); 1.5 (one angle, several distributions)
Insert a step between the worked example and independent practice: model write-up copied out, each proof re-proved on the board by a pupil before any application	Progression by calibrated steps (pu dian): adjusting the distance between prior knowledge and new content, neither a leap nor over-guiding. The fill-in-the-blanks example is the intermediate step, absent from most handouts.	Model write-up copied in S1 before 1.3; 2.2 as model before twins 6.1/6.2; “fill-in” examples in the course notes (numbered bow-tie, reverse direction)
On every geometry problem, require representation before calculation: labelled figure + proportionality table (“pair / set up”) BEFORE allowing any cross-multiplication	[S] Working memory predicts success on word problems; under dual-task conditions, success via the algebraic route drops from 70 % to 40 %, and identifying relations between data is the costliest step. Research on the bar model method ranks missing or poorly represented vital information among recurring errors. [D] “Poor pairing is mistake no. 1” is a local observation: the course notes rank it 2 nd of its six classic mistakes (p. 24).	“Pair / set up / solve” method (S2–S3); sketch required in 4.1 (S5); the Wheel

Transposition or decision in the guide	Finding retained	Concrete trace in this chapter
On true/false items and flawed sample work, have the whole class vote by show of hands BEFORE opening any justification; a numerical counterexample is required for every “false”	Misconceptions must be expressed to be addressed; voting engages the whole class before the explanation. Separating two questions (“the calculation?” then “the presentation?”) is a learning goal.	1.5 (item-by-item vote, S3); 2.3 (“ <i>DE</i> with which side?”, S3); 3.6 (vote on Marie then re-vote, bank R); 5.1
Run through the Problem Wheel: what I KNOW / what I WANT / my TOOLS / my CHECK: before allowing calculation; replace “do you understand?” with “how do you know?”	[S] The “inner teacher”: the pupil’s metacognition is described as an internalised copy of the teacher, acquired in seven steps: metacognitive advice given right when the pupil needs it, success, positive feeling, memorisation, transfer to neighbouring problems. [D] Banning “do you understand?” is a local rule: the research actually makes it the typical monitoring question that the pupil internalises.	Wheel introduced on 4.1 (S5), applicable to 3.6 in bank R; routine no. 5; permanent poster
Include problems where the raw result is NOT the answer: confront a mathematical model with reality, and let the false claim stand before refuting it by calculation	[S] Faced with the absurd problem “there are 26 sheep and 10 goats on a ship: how old is the captain?”, about 90 % of Chinese pupils in 4 th year of primary school, 82 % in 7 th –8 th year and 34 % in Year 13 “solve” it by combining the numbers; many explain that “a problem given by a teacher always has a solution”. [T] Hence problems where the raw result is not the answer, and critical thinking treated as content.	3.5 Statue of Liberty (S5), 3.4 Arthur, 3.6 cones (bank R), 7.3 giant; plausibility required (4.2d, 5.2)
Close every solution with an explicit check: are all three ratios equal? is the longest side opposite the largest angle? order of magnitude?: until the habit becomes a reflex	[S] In the textbooks studied, the looking-back step (checking) is unevenly made explicit. [T] Here the guide makes checking observable in models and pupil work. [D] The routine is kept only if it improves error detection and explanation.	5.2 “check without calculating” (S5); checking the three ratios at the end of each calculation; “Check” step in the course notes examples; routine no. 4
Devote the first ten minutes of the lesson following the test to ONE anonymised real mistake, rephrased and then generalised; code the scripts and require a written test correction in the error log	[S] <i>Jiangping zuoye</i> (Shanghai, 8 th year, geometric proofs): going over homework at the start of class follows three (sometimes four) routine action sequences: review prior content, present the context of the problem or mistake, explain, then summarise and generalise. Script analysis shows that pupils stumbled not over new theorems but over the <i>standards for writing up a proof</i> : which is precisely what the model write-up in S1 targets. [T] The personal error log is a local transposition.	Sheet S7 (codes R/A/K/D/U); first 10 minutes of the lesson after the test; fluency drills in ch. 01B
Only assign for homework what has already been practised in class, and systematically use what comes back: when handed out, every homework assignment states the lesson in which it will be corrected and used	Homework fulfils six functions provided it has public status: assigned on familiar ground, always used on return. A concept discovered alone at home and homework never corrected cancel out the effect.	“Homework” column of each sheet; evening brevet from S3 onwards
Label each exercise by its function (M / R / H), cover drills in class, and provide self-tests with worked solutions that the pupil uses alone before asking for help	[S] Chinese textbooks classify their exercises by function: practice (in-class consolidation), Group A practice “basic, for everyone” and Group B “more demanding, for advanced pupils”, review, self-test (self-check on basic objectives) and reflection question. Self-tests are “left to the pupils themselves”, and only about 40 % of pupils often or always use the answers provided to mark their own work. [T] Hence tracks M and R, modelled on the Group A / Group B pair.	M/R/H tracks on the exercise sheet; microtests + MCQs as self-tests; worked solutions to the exercise sheet handed out on the evening of S5 under a usage agreement
Before the test, run through real exam conditions and have pupils assess sample solutions using the mark-scheme question: “would this paper get full marks?”	Familiarity with item types reduces anxiety; having pupils judge work against a mark scheme is peer formative assessment: assessment literacy is passed on.	6.4 in 10 real min (S6), marking with the grid; evening brevets with the grid; structured announcement of requirements

0 ter. P1–P2 matrix and P9 audit of the chapter

The tables below are audit trails, not quotas: each row refers to an element actually present and makes it possible to observe the level reached.

Concept or method	Reactivated prerequisite	Organised variation (P1/P2)	Observable level
Congruence then similarity	Congruence criteria, angle sum	Congruent triangles, similar non-congruent, right-angled/equilateral counterexamples, rotated figures	S0 reactivation → measured conjecture → S1 proof
Pair and calculate	Proportionality, corresponding angles	Separate, nested, bow-tie; simple scale factor k then less convenient ratio	Worked example → fill-in example in the Course notes → exercise sheet 2.1–2.5

Concept or method	Reactivated prerequisite	Organised variation (P1/P2)	Observable level
Effects of k	Perimeter, area, volume	Length k , area k^2 , volume k^3 , then reverse direction	Guided model → fill-in example on areas → problems 4–5

P9 dimension	Actual element in the chapter	Trace to observe
Concepts	Congruent/similar triangles, corresponding parts, scale factor, scaling effects on dimensions	Course notes parts 1–4; written summaries S0–S4
Techniques	Pair up, set up a table, calculate via k or cross-multiplication, check three ratios	Exercise sheet 1.4, 2.1–2.4; fill-in example
Processes	Conjecture by measuring then prove; compare two methods; model a scale	S0–S2; exercise sheet 5.3 and 6.1–6.3
Metacognition	Say “pair before calculating”, choose the method, check direction and order of magnitude	Guide S2/S5; worked solutions 1.4 and 5.3
Attitudes	Accept that a drawing is not a proof, look for a counterexample, critique a scaling model	Exercise sheet 1.5, Statue of Liberty, bank H

1. Overview

#	Format	Title	Core of the lesson	Homework (assigned → used in)
S0	A	Test 01 feedback + definition	test 01 feedback (0–10), inherited flash, congruence criteria, conjecture activity, definition + corresponding parts, 1.1 started	read part 1 + vocabulary by heart + finish 1.1 + 1.2a + part 1 microtest → S1/S2
S1	A	The Proof	“two angles are enough”, the proof angles ⇒ proportional sides (copy of the small triangle, ASA criterion, corresponding angles, Year 9 nested triangles), 1.3 started	re-read the proof + model write-up + finish 1.3 + 1.2b → S2
S2	B	Calculate: bow-tie + the brevet	timed 1.1/1.2, 1.4 bow-tie (via angles), 2.2 the brevet exercise in depth (pair/set up/solve)	finish 2.1 + 5.1 + microtest → S3
S3	B	Configurations + SAS + vote	2.3 (anti-parallel: the trap), 2.5 in depth, the peak : SAS for similarity + proof by contradiction, 1.5 full vote, 2.4 started	finish 2.4 + microtest; weekend problem (if announced): 6.1 + 6.2 + 5.3 → S4
S4	A	Enlargement k , k^2 , k^3	activity 3.2 (conjecture k^2), proof of k^2 , timed drill 3.1, 3.3 reverse direction	finish 3.3 + microtest + the brevet 6.3 → S5
S5	B	Create + critical eye + the Wheel	5.2 and 3.4 quick-fire, the “modify, create, have a peer solve” cycle on 5.3: the peak, 3.5 statue (critical eye), 4.1 park started (Wheel)	finish 4.1 + selection from 4.2 within the M/R budget: end of scheduled R selection
S6	B	Algorithms and similarity	3.8 Scratch , prediction, individual execution and justification; 6.4 as transfer; test announcement	Summary + MCQ + twin 6.6 (+ 6.5 if second evening) + missed microtests
S7	·	Test (50 min)	current template V3 (Ex. 4 = Proof choice A/B)	test correction required (error log)

After each lesson, the short set on the sheet is assigned in the chosen homework slot, within the budget of section 2: “**Gammes du soir (evening drills)**” (reactivation of ch. 01 + ch. 00 + a Year 9 fluency skill useful for the chapter, answers provided); the weekend follows the announced rotation: a full exam-style problem at most every other weekend, otherwise 30 min of review/error log or rest, cf. the homework policy, part 2.

Pacing target (teacher decision, 2026-07-09): the R track is completed (classwork and homework) by the evening of S5, three quarters of the way through the chapter. The final quarter (S6 and its evening) is reserved for the brevet problems and extension work. If a lesson overruns, cut basic items (drills, warm-ups, reactivations): never R exercises or the brevet quarter.

Official curriculum competencies practised (benchmark for school reports and observations):

Competency	Where it comes into play in the chapter
Investigating	the conjecture activity (S0); searching for the proof (S1); 3.2 (k^2 conjecture, S4)
Modelling	3.5 statue, 4.1 park, 4.2 scale model (choosing k , k^2 or k^3), 3.6 cones
Representing	labelled figures and proportionality table (pair/set up); required sketch in 4.1
Reasoning	the Proof (S1) and the proof of k^2 (S4); 1.5 true/false (counterexamples); 5.2 (invalidate without calculating)
Calculating	drills 1.1 / 3.1; pair/set up/solve (2.2, 2.3, 2.5)
Communicating	model write-up of the proof; the brevet exercises under exam conditions (S6); verbalising invariants (3.2, 1.5)

1 bis. Cross-reference exercise sheet ↔ lessons, and associated fast track

Each exercise led in class is placed in a specific lesson (its “Teacher” note on the teacher version of the exercise sheet gives the teaching notes; the lesson sheet gives the context); the right-hand column says where to send anyone who finishes early right away: always within the same topic, never “wait for the others”.

Exercise (notes: Teacher sheet)	Lesson · phase	For early finishers (in order)
1.1 identify via angles	S0 · ph. 4; timed S2 · ph. 1	1.2 (via sides), then 1.5a (two equilateral triangles: always similar?)
1.3 justify ($\times 3$ configurations)	S1 · ph. 5	the rest of 1.3, then 1.5 c) and e) (preparing for the S3 vote)
1.4 the bow-tie (via angles)	S2 · ph. 3	2.5 ahead of time (the proportional bow-tie, without parallel lines): without telling the others
2.2 the brevet exercise	S2 · ph. 4	the brevet 6.1 (same skeleton): inventing a twin with different values is the next step up
2.3 two configurations	S3 · ph. 2	2.6 (H track, “areas” twin), then an extra numerical counterexample for the vote on 1.5 (5.1 was already assigned for homework on the evening of S2)
2.5 SAS for similarity	S3 · ph. 3	6.9 (the hidden common angle: anti-parallel + reverse direction)
1.5 true/false (vote)	S3 · ph. 4	prepare in writing a different numerical counterexample for each “false”
3.2 conjecture k/k^2	S4 · ph. 2	3.4 (Arthur), then 3.3 a) ahead of time (similarity before area)
3.3 reverse direction	S4 · ph. 5	6.3 (similar triangles and areas), then 7.2 (biology under the microscope: same technique, standard form from ch. 01), then 6.7 (perimeter $\times k$ vs area $\times k^2$, H)
the “modify, create, have a peer solve” block (on 5.3)	S5 · ph. 2	make their own problem harder (remove the superfluous datum, require checking), then have a peer re-solve it: never another chapter
3.5 Statue of Liberty	S5 · ph. 4	7.3 (the impossible giant: the square-cube story, H)
4.1 the scale park (+ Wheel)	S5 · ph. 5	4.2 (the scale model) independently: it is the evening’s homework
3.6 ice-cream cones	Bank R after the core track	6.8 (photo aspect ratios: proportions not preserved, H)
6.4 exam conditions + grid	S6 · ph. 4	the brevet 6.9, question c) “explain why” (waiting quietly)

Permanent fast-track pathway (fast pupils or those ahead on the chapter), rules valid at any time:

- During an investigation phase** (S0, S1, S4): never “move on to the next part”; the instruction is always “find a SECOND justification” or “get ready to write up the proof on the board”. Being ahead is turned into depth, not rushing forward.
- During a drill or a set of exercises:** the right-hand column of the table above, always the next step up on the same topic (1.5, 2.5, 2.6, 3.7, 4.4, 4.5, 6.7–6.9), never another chapter; 2.6 and 3.7 explicitly remain H exercises there.
- From S3 until the test:** the sorted bank (part 7 of the exercise sheet) independently at the back of the classroom, under the working agreement: no help, ungraded, getting stuck is expected. Milestones: 7.5 (re-proving the characterisation, SSS case), 7.6 (the Sierpiński triangle: opening towards upper secondary school fractals), 7.8 (metric relations in a right-angled triangle, Pythagoras re-proved: offer a ten-minute presentation at the board to anyone who completes it).
- Chapter completely finished:** the Brevet H bank in the order 6.7 → 6.8 → 6.9, then the major similarity cascades 4.3 (the geometric mean) and 4.5 (altitudes in action), and 7.7 (the “almost twins” challenge); then the ultimate step, **producing**: inventing a twin of 2.2 with its worked solution, an item of 5.2 (a false result detectable

without calculation), or preparing the pair of presentations $7.8 \rightarrow 7.9$ (Pythagoras re-proved twice), rounded off by 7.10 which compares these proofs with a third one and asks to choose the most enlightening. A pupil who knows how to design the exercise has gone beyond the exercise.

5. **Peer tutoring:** possible but never imposed or by default—a fast pupil is not a teaching assistant; if they help a classmate, it is by asking the questions from the Problem Wheel, not by giving the answers.

2. Chapter equipment and logistics

- **Photocopies:** course notes handout + exercise sheet + “Gammes du soir (evening drills)” sheet handed out in S0 (one per pupil; for the evening drills sheet, a one-sentence reminder of the Charter working agreement—the Charter, handed out and signed in S0 of Ch. 00, is not handed out again); **Summary sheet** handed out in S6 only (“revision object” effect); test paper in S7; worked solutions to the exercise sheet handed out at the end of S5, under a usage agreement: late enough for the techniques to be established in class (the drills and matching are corrected collectively until S5), early enough to equip independent work on the ambitious homework 4.2, the evening Brevet tasks (S3–S6) and the revision weekend. **Worked solutions usage agreement**, stated when handed out and non-negotiable: 1) try and write out your solution first, open the worked solutions afterwards, never the other way round; 2) compare, identify your mistake, and if it is a conceptual mistake it goes into the error log; 3) the “model answer” boxes are used to judge your own writing (“would my paper get full marks?”: assessment literacy); 4) the evening Brevet task is always written with the worked solutions closed, checking afterwards. Rationale: Chinese textbooks provide the answers to self-tests precisely so that the pupil looks for their mistake on their own: independence is built with the worked solutions, not against them; the warning sign remains homework suddenly free of crossings-out.
- **Organisation of pupil materials:** two books + one log. Notebook: the written summary of each lesson, dated, copied from the board in the order of the reasoning—the Japanese board (bansho) is designed to be copied: the written summary tells how we came to know, not just what to know; the handouts are glued into it. Exercise book: evening drills, exercise sheet, evening Brevet tasks—for the evening Brevet tasks, no rough paper: pupils write directly as in the brevet (French national exam at the end of Year 10). Error log (local adaptation of observed practices for reviewing homework; to be assessed on the quality of corrections): after each test and each failed micro-test, the pupil records the problem in one line, THEIR mistake, the correction, and “I got it wrong because...”; it is filled in from the first test of the year (Ch. 00) and re-read before each subsequent one.
- **The board (bansho): standard layout for the year**, announced from S0 and identical in every input lesson, LEFT: the problem, the figure and the given data (and the rough work area, the only erasable area); CENTRE: pupils’ work, displayed in the order chosen while circulating, including incorrect attempts; RIGHT: comparison followed by institutionalisation (the written summary). Three rules of use: 1) nothing is erased before the summary, the board is the collective memory of the reasoning; in geometry, the marked figure remains displayed as long as the proof is not closed; 2) delayed copying: during the investigation and whole-class discussion pupils do not copy, they think; they only copy the summary; 3) **representations are only valuable when linked:** a figure and a proportionality table side by side only help if the link is drawn (matching arrows angle $\rightarrow$ corresponding side). **Ambition rule (2026-07-10):** the CENTRE column stages the peak of the lesson, the most demanding exercise conducted collectively (the conjecture, the Proof, 2.2, 2.5, the k^2 , the “modify, create, have others solve” cycle, the cones 3.6): from its problem statement to its solution, including crossed-out traps; drill, fluency skills and quick corrections are restricted to the LEFT CORNER; only the test and test feedback are exempt from the rule. The board plan for each input lesson is prepared before the lesson (bansho keikaku). The ideal boards in this guide also exist in large format in the separate document TableauxProjection (landscape, one board per page): to be projected as a summary check or backup; the live chalk board remains the norm.
- **Configuration gallery** (A3 poster or permanent corner of the board): as they are encountered, pin up the marked standard figures: bow-tie / hourglass (S2), altitude configuration (mentioned in S5/part 6), antiparallel (S3). A recognised configuration is half solved.
- **Mini-whiteboards** (or half-sheets): essential for the exit flashes in S0, S1, S3, S4, the physical setup for “vote before justifying”.
- **Visible timer** for timed drills (1.1: 6 min; 3.1: 8 min) and for the 10 min “brevet conditions” of 6.4 (S6). **Graduated ruler / set square** for the measuring activity in S0 and constructions (3.2).
- **“Problem Wheel” poster** (A3 or drawn on the board) from S5 onwards: it will remain on display all year.
- **Teacher’s logbook:** note down in each lesson the 2–3 mistakes actually observed (they feed into the next fluency session and the test feedback).
- Observable homework budget [D]: local decision to be evaluated over four weeks. The three tiers remain Learn (active re-reading), Evening drills (self-corrected reactivation) and Do (already practised). A lesson evening aims for 45 min of effective work and never exceeds the 60 min red line. Track *M* (core track) aims for 35–40 min; stopping at the time limit is allowed with a note at the last question completed and is never penalised. Track *R* (recommended track) aims for 40–50 min: this is the test level. Track *H* (going further) is voluntary, replaces a task and is never added on top of it. No leftover work spills over to the next evening and no exercise is added to “catch up”. At the weekend, a

full practice paper under exam conditions is offered at most every other weekend; the alternate weekend switches to *30 min of review/error log or rest*, as announced by the teacher. The “End of week” sheets give the content of the paper weekend when activated, never a second cumulative workload. For four weeks, the logbook records the actual duration (M/R), stops at the time limit, any approach to the red line and two pupil feedbacks; the workload is then kept, lightened or shifted. Research supports the value of review and error analysis; this numerical calibration remains a local decision to be tested.

- **Weekly reactivation flash:** routine carried over from Ch. 00, nothing new to set up: 5 min at the start of the first lesson of the week, ONE homework exercise from the past week used as a reference structure, with data possibly changed, formative by default; frequency depends on the observed goal; any grading is announced in advance, with criteria and an opportunity to redo; on the day of the flash, it replaces the opening fluency task. The exercise is always designated in the previous end-of-week sheet. For this chapter (2 weeks): week 1 → flash inherited from the previous chapter: Ch. 01 exercise sheet, the first two questions of exercise 5.12 (Brevet workshop: fractions and standard form), used as a reference structure, with data possibly changed (5 min format), given at the start of the first lesson of the chapter; it is chosen from the homework of the last week of Ch. 01 (the twin exercises 5.10/5.12/5.14 on its S8 sheet share the same skeleton: the pupil encounters a familiar format, whichever twin was written up in track M); week 2 → **1.3 b)** (designated on sheet S3, given at the start of S4); sheet S7 designates the exercise passed on to week 1 of chapter 01B (3.3 b–d).
- **Rotating collection:** in place since Ch. 00, simple continuation: 3–5 exercise books checked in each lesson, on a rota (each book is checked roughly every two weeks); what is looked at is the evidence of working and investigation, never neatness.

3. The S0 diagnostic, item by item

The 10 fluency skills in the handout (course notes, p. 2) serve as a silent X-ray. Items 5 to 10 are the direct prerequisites for the chapter (angles, congruence criteria, proportionality, scale, effect on areas). Reading the errors:

Failed item	What it indicates	Immediate remediation
1–4) simplify, $\frac{2}{3} \times \frac{9}{4}$, squares/cubes, prime factorisation	numerical toolkit for the chapter (ratios, k^2 , k^3) is fragile	the evening drills address this in the selected homework slots (one question from Ch. 01 per set, sheet handed out in S0); call on these pupils first for 3.1
5) 3 rd angle (54° , $81^\circ \rightarrow 45^\circ$)	angle sum = 180° not mobilised → “two angles are enough” concerns them first and foremost	call on them first during the S1 whole-class discussion; S0 exit flash (the 47/86)
6) right-angled triangle with a 25° angle: other two angles?	link angles → shape not automatic	revisited through 1.1 in class (S0); note down the names
7) state the three congruence criteria (Year 9)	prerequisite for the Proof (the ASA case is at its heart)	table of the three criteria in the course notes (part 1) to be recited at the start of S1; warning threshold
8) 8-15-17 triangle (converse of Pythagoras)	tool needed in several exercises (2.4, 4.1) is missing	used in homework 2.4 on the evening of S3; refer back to the Year 9 review chapter
9) scale 1/250: what does 1 cm represent?	concept of scale not secure	will be revisited in 4.1 (S5); note down the names
10) 3×5 rectangle, dimensions doubled: area?	the $\times k$ instead of $\times k^2$ error is already present	this is the focus of S4: before/after witness; use it as a hook

Warning threshold: a pupil who fails item 7 (congruence criteria) and item 5 (3rd angle) requires close monitoring—the Proof in S1 will be opaque to them without individual reactivation (see section 9); write down the list right from S0.

4. Detailed lesson guides

S0: Year 9 reactivation and discovery (format A, 55 min)

Pupil objective. Reactivate the congruence criteria (Year 9); discover that two triangles can have the same shape without having the same size, and conjecture the proportionality of the sides. **Skills:** investigate, calculate.

Kadai (to be written on the board, left zone, boxed). “Do two triangles with the same angles always have proportional sides?”: the lesson summary (matome) must answer this question; this is the overarching challenge behind the measuring activity.

Equipment. Course notes handout + exercise sheet + “Gammes du soir (evening drills)” sheet handed out; graduated ruler; the table from section 3 of this guide at hand.

Orchestrating the transition. Returning test 01 and the inherited flash quiz are included within the 55 minutes below: no hidden time before S0. Prepare the anonymised error and the F/P/N/U/R codes before the pupils enter.

Phase 1 (0–10): Test 01 feedback, genuinely integrated

Hand back the papers; project ONE real anonymised error. Two pupils rephrase it, the class corrects it and then generalises it. Each pupil opens the corresponding entry in their error log; announce the 2–3 dominant F/P/N/U/R codes and how they will be revisited. At 10 minutes, papers are put away; the full individual correction continues at home.

Phase 2 (10–15): Inherited flash quiz from chapter 01

Same procedure and same structure as the first two questions of the Ch. 01 exercise sheet, exercise 5.12, with data possibly changed (5 min, formative by default). Collect at 15 minutes; if grading was announced in advance, apply the published criteria and allow a retake. No additions outside the time budget.

Phase 3 (15–26): Reading agreement and targeted diagnostic

Handouts are distributed upon entry. Remind the class of the working agreement in one sentence, then read only the “Starting point” box (course notes, p. 1). The 10 fluency skills are completed in 8 min; 2 min targeted check on items 5, 7 and 10, with the other answers displayed. Quietly tick the table in section 3.

- Q1.** “In this handout, what will be required in the test, and what counts as going further or an accepted result?”, Goal: establish the reading agreement from day one. Expected: required = the definition, the angle characterisation AND ITS PROOF, the “match / set up / solve” method, the k , k^2 , k^3 effects; going further / accepted = the converse of the characterisation (p. 9) and the 3rd criterion “SAS for similarity” (part 2.3). Check: have pupils point to one box of each category.
- Q2.** “What are the three icons on the exercise sheet for?”, institutionalise the exercise sheet key: **M** core track (for everyone), **R** recommended track (test level), **H** going further (optional: the bank). Then the 10 fluency skills (course notes, answers upside down at the bottom of the box), individually, 8 min, no calculator: targeted show-of-hands check (items 5, 7 and 10) without detailed correction; quietly tick the table in section 3 (especially who fails items 5, 7, 10).

Phase 4 (26–31): Reactivation, the three congruence criteria

The congruence criteria are **prior knowledge from Year 9**: reactivation, not rebuilding. Table of the three criteria on the board (course notes part 1, p. 2).

- Q3.** “State the three congruence criteria for triangles.”, Goal: reactivate THE prerequisite for tomorrow’s proof (the ASA case). Expected, required word for word: “side-side-side” (all three sides); “side-angle-side” (an angle INCLUDED between two sides); “angle-side-angle” (a side INCLUDED between two angles). Follow-up if “angle-angle-angle” is mentioned: “three equal angles: are the triangles necessarily CONGRUENT?”: no, and that isatus/exactly what we are going to discover.

Quick answers (teacher): Target misconception to leave alive: “angle-angle-angle is a congruence criterion”, put it on the board, keep it to invalidate at the end of the lesson (it is the gateway to similar triangles).

Phase 5 (31–47): Conjecture activity (hatsumon, kikan-shidō, neriage)

Draw on the board (or project) the two triangles from the activity (course notes, p. 6), with angles 65° , 45° , 70° and

different sizes. Instruction: “Measure the six sides to the nearest millimetre, line them up opposite the equal angles, calculate the three quotients.” Individual work 5 min, pairs 3 min, selective whole-class discussion 8 min. Circulate and select three contrasting pieces of work, including one “random” comparison.

Pre-coded seating plan (standard kit, part 5 bis). Circulate with the annotated seating plan (A/B/C/**X**) to cast the speaking order for the neriage rather than improvising it.

Q4. “What do you notice about the three quotients? State a conjecture: ‘If two triangles have pairs of equal angles, then...’”, Goal: bring out proportionality through observation, before naming it. Expected: the three quotients are (almost) equal (here $\approx 1,5$) so “... then their sides are proportional”. Follow-up if the quotients differ: “did you compare the side opposite 70° in one with the side opposite 70° in the other?”.

Q5. “Why did we line up the sides opposite the equal angles, and not at random?”, Goal: introduce the word CORRESPONDING and the “matching” habit before any calculation. Expected: those are the sides that match up (corresponding sides); opposite equal angles lie corresponding sides. Follow-up: display a “random” piece of work (e.g. AB/BC) and have the class see that the ratios are no longer equal: “we didn’t match them up”.

Anticipated pupil responses (order of presentation):

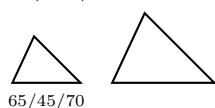
1. **X** sides compared “in alphabetical order” ($AB/A'B'$ but also $AB/B'C'$): unequal ratios, keep for Q5;
2. **X** “the triangles are congruent” (confusing congruent/similar): mentally superimposing them fails, different sizes;
3. $\checkmark$ quotients $\approx 1,5$ lined up opposite the angles: the conjecture;
4. $\checkmark$ “it’s an enlargement with scale factor 1,5” (vocabulary from part 4 anticipated: praise it).

IDEAL BOARD (final state): to project or reproduce

LEFT: the activity

Kadai: “Same angles $\Rightarrow$ proportional sides?”

Two triangles with the same angles $65^\circ/45^\circ/70^\circ$, different sizes:



Measure the 6 sides, compare opposite equal angles.

CENTRE: pupils’ work

At random ($AB/B'C'$): ratios $\neq$ **X** (not matched)

Opposite equal angles:

$$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC} \approx 1,5$$

Conjecture (pupils): “pairs of equal angles $\Rightarrow$ proportional sides”

But three measurements prove nothing: to be proved tomorrow.

RIGHT: definition (the summary)

Similar = proportional sides.

k : scale factor.

Corresponding sides / vertices = matching ones (opposite equal angles).

Tracks: M core / R recommended / H going further.

Phase 6 (47–55): Partial institutionalisation + launch of drill 1.1

Co-constructed matome (standard kit, part 5 bis). Before copying the written summary: re-read the kadai, have the class formulate the answer in one sentence (written in the right-hand zone in their own words), then compare with the handout. The written summary below remains unchanged.

TO COPY INTO THE NOTEBOOK (word for word)

Similar triangles. Two triangles are similar when the lengths of their sides are proportional. The constant k that takes the sides of one to the sides of the other is the scale factor. The sides (and vertices) that match up are called **corresponding**: opposite equal angles lie corresponding sides.

Conjecture of the day (to be proved): “if two triangles have pairs of equal angles, then their sides are proportional”.

Warning: three measurements prove nothing; this conjecture is not yet proved. We will prove it in the next lesson.

Drill 1.1 (M): two items on the board (recognising by angles, calculating the third angle); the next two move to the homework already planned for the evening. Then **step-up**:

Q6. “Are two equilateral triangles always similar?” (1.5a), Goal: first encounter with “similar without being congruent”. Expected: yes, their angles are all 60° , so equal in pairs; but their sizes may differ ($k \neq 1$). Follow-up: “what about two right-angled triangles?” (no, not necessarily: the other two angles vary).

Differentiation. Fragile pupils: stay on 1.1 (calculating the third angle). Fast pupils: 1.5 c) (two right-angled triangles) and e) (the isosceles triangle with an 80° angle).

Homework. Learn: read part 1 (congruent triangles) and the beginning of part 2 (course notes, p. 2 to 6). • By heart: the vocabulary—superimposable, congruent triangles, congruence criteria, similar triangles, corresponding, scale factor. • Evening drills: “Evening of S0” block on the sheet (5 questions, answers at the bottom of

the block). • Do: finish **1.1 + 1.2 a)** + the micro-test in part 1 as a self-test (course notes, p. 5: congruence criteria, the prerequisite for tomorrow's Proof) + redo in writing the failed items from the diagnostic (course notes, p. 2). Track M (stop at 35–40 min): evening drills + end of 1.1 + 1.2 a) done with the table of the three criteria and the worked example from the course notes in front of them + the part 1 micro-test + failed diagnostic items (scaffolded support intended). Track R (40–50 min): add a written numerical counterexample for 1.5 c) (two right-angled triangles).

ÉCOLE DIRECTE (school messaging platform): text ready to paste

Read the course notes: congruent triangles, then the definition of similar triangles.

Learn by heart: superimposable triangles, congruent triangles, congruence criteria, similar triangles, corresponding vertices and sides, scale factor.

Evening drills: “Evening of S0” block on the sheet; answer before reading the answers.

Do: 1.1, 1.2 a) and the flash questions in the course notes.

Rework: the failed fluency skills from the entry test, course notes.

Bring: the course notes, the exercise sheet and the evening drills sheet.

Optional: write a numerical counterexample for 1.5 c), “two right-angled triangles”.

If running late. Skip the step-up 1.5a (pick it up as the opening flash in S1); the conjecture activity and the definition cannot be cut.

Success criteria... exit flash on mini-whiteboards “ 47° and 86° here, 47° and 47° there: similar?”: more than 70% answer yes (third angles 47° and 86° : same three angles)—a trap, not rote recitation; and the list of warning-threshold pupils (items 5 and 7) is drawn up.

Teaching note: Explicit reading agreement and exercises labelled by function. The measuring conjecture activity opens the problem-based lesson: Japanese-style devolution. Conjecture is not proof, doubt is established at the end of the lesson: it will be resolved by the proof in S1 (delayed matome). The misconception “angle-angle-angle = congruence” is elicited and then turned into the gateway to similar triangles (focusing questioning).

S0 (variant): phase 5 “two given angles, free sizes” (alternative script)

Status. Variant of phase 5 of S0 (Japanese lesson on constructing congruent triangles, transposed to similarity). **The default activity** (measuring the two projected triangles) remains the standard route; this variant replaces it on days when rulers + protractors are available and you want the pupils to produce the triangles themselves: it is a type 4 neriage (sorting an open problem), otherwise missing from the set. The definition and the written summary (phase 6) are UNCHANGED.

Kadai (identical to S0). “Do two triangles with the same angles always have proportional sides?”

Task calibration (four criteria). The requested triangle must: (1) fit in the notebook; (2) be visible from afar once displayed; (3) have sides measurable to the nearest tenth of a cm; (4) have integer angles. Hence the choice: two given angles 35° and 65° (integers; third angle 80°), the length of one side free.

Phase 5 variant (31–47): produce, display, sort (hatsumon, kikan-shidō, type 4 neriage)

1. Instruction (hatsumon). “Each pair constructs a triangle with a 35° angle and a 65° angle; you choose the length of the side between these two angles. Write down your three measured sides.” (ruler + protractor.) **2. Circulating (kikan-shidō).** Spot triangles of very different sizes; set aside 3–4 contrasting ones for display (pre-coded seating plan, part 5 bis). **3. Display.** Put all the triangles up on the board: a gallery.

Q4’. “Sort these triangles into families: what brings them together, what sets them apart?”, Goal: have the class name the sorting criterion (that criterion is the mathematical content). Expected: a single family, “same shape”, different sizes; duplicates (same dimensions) are removed. Follow-up: “superimpose two of them: can you do it?” (no: different sizes).

Q5’. “Measure the sides of two very different triangles, and compare them opposite the equal angles.”, Goal: bring out proportionality. Expected: the side/corresponding side quotient is the same for all three pairs $\Rightarrow$ proportional sides; the angle characterisation emerges from the sorting. Follow-up: “line up the sides opposite the equal angles before dividing”.

Q6’. **Live counterexample.** “What if we only fix a single angle (35°)? Quickly draw another triangle.”, Goal: show that two angles are necessary. Expected: the resulting triangles are no longer all similar (the other two angles vary): one angle is not enough. This counterexample prepares mistake no. 4 in the course notes and the vote in 1.5.

Anticipated pupil responses (order of presentation):

- ✓ triangles of varied sizes, all 35/65/80: the gallery (generic case);
- ✗ “they are all congruent” (confusing congruent/similar): superimposition impossible;
- ✓ constant ratios opposite equal angles: the conjecture;
- ✗ (only one angle fixed) non-similar triangles: the counterexample.

IDEAL BOARD (final state): to project or reproduce

LEFT: the open task

Kadai: “Same angles $\Rightarrow$ proportional sides?”

Construct a triangle: angles 35° and 65° fixed, side free.

Calibration: fits in the notebook · visible from afar · sides to the nearest tenth · integer angles.

CENTRE: the sorted gallery (type 4 neriage)

Gallery: all triangles 35/65/80, same shape, different sizes.

✗ “they are congruent”: superimposition impossible (sizes $\neq$)

Sorting (class): a single family; named criterion: same angles

Measurements: $\frac{\text{side}}{\text{corresponding}}$ constant $\Rightarrow$ proportional sides

Counterexample: only one angle fixed $\Rightarrow$ non-similar triangles

RIGHT: definition (written summary, unchanged)

Similar = proportional sides.

k : scale factor.

Corresponding sides / vertices (opposite equal angles).

Two angles are enough; one alone is not enough.

Next. Phase 6 (institutionalisation + drill 1.1) is identical to the default route. This variant feeds directly into 1.1 (“Similar or not? Using angles”) and 7.5 (the converse of the characterisation).

Teaching note: Task of constructing congruent triangles, transposed here to similarity. Type 4 neriage: classify pupils’ constructions by angles, then bring out the invariant and the relationship between corresponding sides.

S1: The Proof (format A, 55 min: key lesson of the chapter)

Pupil objective. Understand why equal angles lead to proportional sides, and know that two angles are enough. This is the candidate proof for the “Proof” question on the test. **Skills:** reason, investigate, communicate.

Kadai (to be written on the board, left zone, boxed). “How can we prove (and no longer merely observe by measuring) that equal angles lead to proportional sides?”: the matome answers this.

Equipment. Course notes part 2; exercise sheet 1.3; mini-whiteboards; the table of the three congruence criteria displayed.

Phase 1 (0–10): Fluency + targeted reactivation

Fluency: 4 items “third angle / similar?” (1.1).

Quick answers (teacher): $180 - 47 - 86 = 47$; $180 - 92 - 44 = 44$; two equilateral triangles: yes; two right-angled triangles: not necessarily.

Reactivation geared towards the proof: recite the three congruence criteria, emphasise **ASA**.

Q1. “Yesterday’s conjecture: pairs of equal angles $\Rightarrow \dots$? And are three measurements enough to prove it?”, Goal: reopen the doubt established in S0. Expected: “ $\Rightarrow$ proportional sides”; no, measuring = conjecturing, not proving. Follow-up: “how many 65/45/70 triangles could I draw? Infinitely many: we cannot measure them all”.

Phase 2 (10–16): “Two angles are enough” (express matome)

Q2. “Why are two pairs of equal angles enough, without looking at the third?”, Goal: establish the lemma that streamlines all the proofs in the chapter. Expected, required word for word: “because the sum of the angles in a triangle is 180° ; the third is 180° minus the other two, so it is determined: $\widehat{C} = 180^\circ - \widehat{A} - \widehat{B} = 180^\circ - \widehat{A'} - \widehat{B'} = \widehat{C'}$ ”. Follow-up if stuck: “what is $\widehat{C}$ if $\widehat{A}$ and $\widehat{B}$ are known?”.

Phase 3 (16–40): Building THE proof (hatsumon, neriage of a proof)

Write on the board: **Given** $\widehat{A} = \widehat{A'}$, $\widehat{B} = \widehat{B'}$; **Goal:** $\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC}$. First handle the case $A'B' = AB$ using ASA ($k = 1$), then, swapping the roles of the two triangles if necessary, assume $A'B' < AB$: it is this hypothesis that makes it possible to fit a copy of the small triangle inside the large one. The question: “How can we PROVE this using what we know from Year 9 and Year 8?” Individual/pair investigation 5 min. Circulate, choose the order of presentation.

Anticipated pupil responses (order of presentation):

1. ✗ “we measure the three sides and check”, display it without comment, then: measuring $\neq$ proving;
2. ✗ “the triangles are congruent”, no: $k \neq 1$ in general;
3. ✓ key idea: “place a copy of the small triangle inside the large one”;
4. ✓ key idea: “a line parallel to (BC) ” / “the nested triangles from Year 9”.

Q3. “When can we place a copy of the small triangle INSIDE the large one? Where should we place point M ?”, Goal: make the construction condition explicit (step 1). Expected: if $A'B' = AB$, ASA concludes directly; otherwise we swap the triangles if needed so that $A'B' < AB$, then place M on $[AB]$ such that $AM = A'B'$ and draw the line parallel to (BC) through M , which meets $[AC]$ at N . Follow-up: “what should we do if the side to be marked off is longer than $[AB]$?”.

Q4. “Why is triangle AMN congruent to $A'B'C'$? Which congruence criterion?”, Goal: the logical heart (step 2). Expected: $\widehat{MAN} = \widehat{B'A'C'}$ (hypothesis); $AM = A'B'$ (construction); $\widehat{AMN} = \widehat{ABC}$ (corresponding angles, $(MN) \parallel (BC)$ cut by (AB)) and $\widehat{ABC} = \widehat{A'B'C'}$ (hypothesis), so $\widehat{AMN} = \widehat{A'B'C'}$; side $[AM]$ is INCLUDED between the two angles $\Rightarrow$ **ASA** case $\Rightarrow AMN = A'B'C'$, hence $AN = A'C'$ and $MN = B'C'$. Follow-up: “what are the parallel lines used for? Which angles become equal?”.

Q5. “How do we harvest the proportionality now?”, Goal: closing the proof (step 3). Expected: $M \in [AB]$, $N \in [AC]$, $(MN) \parallel (BC) \Rightarrow$ proportionality of nested triangles (the intercept theorem (Thales’ theorem in France) from Year 9): $\frac{AM}{AB} = \frac{AN}{AC} = \frac{MN}{BC}$; substituting $AM = A'B'$, $AN = A'C'$, $MN = B'C'$: $\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC}$. ■ Follow-up: “what are the lengths AM , AN , MN equal to?”.

IDEAL BOARD (final state): to project or reproduce

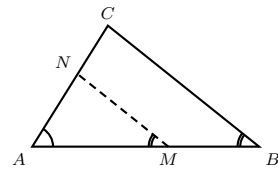
LEFT: given data and figure

Kadai: “How to prove it (not just measure it)?”

Given: $\hat{A} = \hat{A}'$, $\hat{B} = \hat{B}'$.

Goal: $\frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC}$.

If $A'B' = AB$: ASA, so $k = 1$. Otherwise, swapping the triangles if needed, assume $A'B' < AB$.



M such that $AM = A'B'$; $(MN) \parallel (BC)$.

CENTRE: the constructed proof

✗ “measure the 3 sides”: measuring is not proving

1. $M \in [AB]$, $AM = A'B'$; parallel to $(BC) \rightarrow N$

2. $\widehat{AMN} = \widehat{ABC}$ (corresponding) = $\widehat{A'B'C'}$; with $\hat{A} = \hat{A}'$ and $AM = A'B'$:

ASA case $\Rightarrow AMN = A'B'C'$
so $AN = A'C'$, $MN = B'C'$

3. nested (Year 9): $\frac{AM}{AB} = \frac{AN}{AC} = \frac{MN}{BC}$
 $\Rightarrow \frac{A'B'}{AB} = \frac{A'C'}{AC} = \frac{B'C'}{BC}$ ■

RIGHT: summary (written summary)

Characterisation. Two triangles are similar if, and only if, their angles are equal in pairs. **Two angles are enough** (sum = 180°). The direction angles $\Rightarrow$ sides is proved here; the converse is proved in the course notes under going further (exercise sheet 7.5).

Standard wording (D2). “We know that $\hat{\dots} = \hat{\dots}$ because... Similarly, we know that $\hat{\dots} = \hat{\dots}$ because... Now, if two triangles have two pairs of equal angles, then they are similar. Therefore triangles ... and ... are similar, with corresponding vertices ... $\leftrightarrow$...”

Phase 4 (40–48): Institutionalisation (matome), the written summary = the proof

📎 TO COPY INTO THE NOTEBOOK (word for word)

Angle characterisation. Two triangles are similar if, and only if, their angles are equal in pairs. **Two pairs of equal angles are enough** (the third follows, because the sum of the angles is 180°). The direction “angles $\Rightarrow$ proportional sides” has just been proved; the converse direction is proved in the course notes, under going further (exercise 7.5 on the exercise sheet).

Standard wording (model D2 from the course notes, to be copied as is on a test paper): “We know that $\hat{\dots} = \hat{\dots}$ because... Similarly, we know that $\hat{\dots} = \hat{\dots}$ because... Now, IF TWO TRIANGLES HAVE TWO PAIRS OF EQUAL ANGLES, THEN THEY ARE SIMILAR. Therefore triangles ... and ... are similar, with corresponding vertices ... $\leftrightarrow$..., ... $\leftrightarrow$..., ... $\leftrightarrow$...”

Phase 5 (48–55): 1.3 justifying similarity (the three sources of equal angles)

1.3 worked on the board: a) as a whole class (model write-up word for word: common angle + corresponding angles); b) and c) started in pairs; in c), the crossed matching ($E \leftrightarrow B$, $D \leftrightarrow C$) is THE key point: have pupils verbalise “I follow the angles, not the position of the letters”. b), c) are left for homework: the exercise has been started.

Quick answers (teacher): Three sources of equal angles to establish: common angle; vertically opposite angles; alternate interior / corresponding angles (parallel lines).

Differentiation. Fast finishers: prepare in writing the justification for 1.5 c) and e) (for the S3 vote); for the keenest pupils: tackle **7.5**: re-prove the converse (the 2nd case of similarity, SSS), using the same skeleton as today’s proof.

Homework. Learn: re-read the proof and the model write-up (course notes, pp. 9–11). • By heart: “two angles are enough” and why (sum = 180°), be able to explain it. • Evening drills: “Evening of S1” block on the sheet (answers at the bottom of the block). • Do: finish **1.3** (b and c, a done in class) + **1.2 b)** (critical check using all three ratios: this feeds into the error analysis in S2) + the Part 2 microtest as a self-test (course notes, p. 12). Track M (stop at 35–40 min): drills + 1.3 a) rewritten from memory then b), written out with the model write-up copied at the top of the page + the first item of 1.2 b) + the Part 2 microtest (deliberate scaffolding). Track R (40–50 min): add the written preparation of 1.5 c) and e) for the S3 vote. Optional Track H, as a replacement: **5.4** (the two midpoints of the square).

ÉCOLE DIRECTE (school messaging platform): text ready to paste

Read the course notes: the proof and the model write-up.

Learn by heart: two pairs of equal angles are enough, because the third angle is determined by the 180-degree angle sum.

Evening drills: “Evening of S1” block.

Do: 1.3 b) and c), 1.2 b) and the quick-fire questions in the course notes.

Rework: 1.3 a) by rewriting the model write-up from memory.

Optional: 5.4, “the two midpoints of the square”.

Test: this proof may be asked.

If running late. 1.3 becomes homework in full (the model write-up on the board for a) cannot be cut); steps 1–3 of the proof are never cut.

Success if... exit check on mini-whiteboards “Two triangles each have a 40° angle. Similar?”: record the number of “no” answers correctly justified by “two pairs of angles are needed”; schedule a follow-up depending on the observed distribution.

Teaching note: Lesson = Japanese problem-solving lesson applied to constructing a PROOF (hatsumon → kikan-shidō → neriage → matome). The proof assembles prior knowledge built in stages, ASA case (Year 9), corresponding angles (Year 8), nested triangles (Year 9): stepped progression (pu dian). “We measure the three sides” is displayed and then ruled out: measuring is not proving (focusing questioning).

S2: Calculating lengths, the butterfly and the Brevet question (format B, 55 min)

Pupil objective. Calculate lengths using the match / set up / solve method; recognise the butterfly configuration.
Skills: calculating, modelling, communicating.

Kadai (to write on the board, left zone, boxed). “How can we calculate an inaccessible length from two similar triangles?”: the matome answers this (match / set up / solve).

Materials. Exercise sheet 1.1, 1.2, 1.4, 2.2; corresponding teacher instructions; timer.

Phase 1 (0–10): Timed fluency 1.1 (6 min) + targeted review of 1.2b

6 min on the timer, no calculator, all 6 items (recognising via angles). Quick correction: only a) and e) are written in the left corner of the board; a) is the chapter’s trick question (47/86 vs 47/47, still similar); e) requires decoding “isosceles at D ” (main vertex D , 92° angle: base angles 44°). Then review 1.2b, set for the evening of S1: the first two ratios are equal, but the third is not.

Quick answers (teacher): 1.2b: $\frac{1}{5} = \frac{1.6}{8} = 0.2$, whereas $\frac{1.2}{6.5} \approx 0.185$; the triangles are not similar.

Q1. (on 1.2b) “Are two equal ratios enough to conclude ‘similar’?”, Goal: anchor “with sides alone, compare all three ratios”. Expected: no; 1.2b is the exact counterexample. The 3-4-5 / 4-5-6 triple is not used here, as all three of its ratios are different.

Phase 2 (10–16): Using a real homework error

Choose ONE error observed in 1.1/1.2 (typical: concluding “not similar” without calculating the third angle). Anonymise it on the board; vote “right or wrong?” before justifying.

Q2. “What is missing from this reasoning to reach a conclusion?”: have another pupil rephrase (“we need the third angle / all three ratios”).

Phase 3 (16–30): 1.4 the butterfly, using angles

Intercept theorem forbidden (Chapter 01C): here we prove it using angles. Diagram on the board, name the two triangles (ABC at the top, ADE at the bottom).

Q3. “Two pairs of equal angles in the butterfly: which ones, and why?”, Goal: identify BOTH sources of angles independently. Expected: $\widehat{BAC} = \widehat{DAE}$ (vertically opposite angles) and $\widehat{ABC} = \widehat{ADE}$ (alternate interior angles, $(BC) \parallel (DE)$ cut by (BD)); therefore similar. Prompt: “cover the bottom of the diagram: what do you see at A ?” then “what are the parallel lines in the problem used for?”.

Q4. “Match the sides, then calculate AE and BC .”, Goal: match before calculating, and get a feel for the direction of k . Expected: $k = \frac{AD}{AB} = \frac{4.8}{3.6} = \frac{4}{3}$; $AE = 2.4 \times \frac{4}{3} = 3.2$; $BC = DE \div k = 5.2 \times \frac{3}{4} = 3.9$. Error to elicit: $BC = 5.2 \times \frac{4}{3} \approx 6.9$, display it, have pupils rule it out by checking for sense (BC is in the smaller triangle, so it must be shorter than DE).

Phase 4 (30–52): 2.2 the Brevet question, in depth (the climax, the reference solution)

This is the model exercise for official expectations: every pupil leaves with the fully written-out solution (S2 written summary), which will serve as a model for Brevet questions 6.1–6.2 and the test. Diagram copied and marked. The neriage compares four states of the same piece of work: P1 concludes after only one pair of angles; P2 justifies both pairs but matches sides using the letter order; P3 calculates correctly without checking; P4 is the complete solution. The board thus tells the story of 2.2 from problem statement to check; the drill and the butterfly remain stepping stones on the left.

Q5. “Question a: show that $\widehat{BCD} = \widehat{CAE}$.”, Goal: recognise corresponding angles. Expected: $(AE) \parallel (CD)$ cut by (AB) at A and $C \Rightarrow \widehat{CAE} = \widehat{BCD}$ (corresponding angles). Prompt: “which lines are parallel? which line cuts them? what do we call two angles in this position?”.

Q6. “Question b: conclude ‘similar’”, Expected: $(CE) \parallel (BD) \Rightarrow \widehat{ACE} = \widehat{CBD}$; two angles are enough; correspondence $A \leftrightarrow C$, $C \leftrightarrow B$, $E \leftrightarrow D$. Prompt: “is a single pair of angles enough?” (no: error n°4).

Q7. “Question c: the three steps.”, Goal: match / set up / solve neatly. Expected on the board: $CB = 7.5 - 3 = 4.5$ calculated FIRST (many forget this); table set up; $k = \frac{3}{4.5} = \frac{2}{3}$; $AE = 6$, $CE = 3.6$; check of all three ratios. Prompt: “what do we need before setting up the table?” (a complete column: AC and CB).

Anticipation (2.2). **P1** ✗ conclusion after question a alone; **P2** ✗ matching $[AC] \leftrightarrow [CD]$ “in letter order”; **P3** ✓ $CB = 4.5$, table and calculations, without checking; **P4** ✓ complete write-up with three equal ratios. P4 is

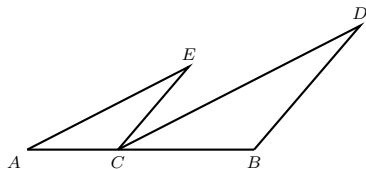
essential, not optional.

IDEAL BOARD (final state): to project or reproduce

LEFT: problem 2.2 + drill corner

Kadai: “Calculating an inaccessible length using similarity”

$C \in [AB]$; $AC = 3$; $AB = 7,5$; $BD = 5,4$; $CD = 9$; $(AE) \parallel (CD)$; $(CE) \parallel (BD)$.



Drill corner: 1.1: a) $47/86 \rightarrow 47$: similar ✓; e) isosceles 92° : base 44° .

1.4 butterfly: vertically opposite + alternate interior; $k = \frac{4,8}{3,6} = \frac{4}{3}$; $AE = 3,2$; $BC = 3,9$; ✗ $BC \approx 6,9$ (impossible: $BC < DE$), crossed out.

CENTRE: 2.2, the journey to the climax

a) $(AE) \parallel (CD)$ cut by $(AB) \Rightarrow \widehat{CAE} = \widehat{BCD}$ (corresponding)

b) $(CE) \parallel (BD) \Rightarrow \widehat{ACE} = \widehat{CBD}$; two angles are enough $\Rightarrow$ similar ($A \leftrightarrow C$, $C \leftrightarrow B$, $E \leftrightarrow D$)

✗ $AB = 7,5$ put into the table (CB forgotten): crossed out

✗ matching “in letter order”: crossed out

c) ✓ $CB = 7,5 - 3 = 4,5$ (first!)

CBD	$CB = 4,5$	$CD = 9$	$BD = 5,4$
ACE	$AC = 3$	$AE = ?$	$CE = ?$

$k = \frac{3}{4,5} = \frac{2}{3}$; $AE = 6$; $CE = 3,6$

check: $\frac{3}{4,5} = \frac{6}{9} = \frac{3,6}{5,4} = \frac{2}{3}$, complete write-up copied (the reference)

RIGHT: summary (written summary)

Match: using the angles (never the letters). Opposite the largest angle lies the longest side.

Set up: table of corresponding sides, starting triangle on top, target triangle on the bottom.

Solve: find k from a complete column, then $\times k$ towards the target triangle; check the ratios (model L1).

TO COPY INTO THE NOTEBOOK (word for word)

Calculating lengths: the three steps. **Match** corresponding sides using the angles (opposite equal angles lie corresponding sides; opposite the largest angle lies the longest side). **Set up** the table of corresponding sides (starting triangle on top, target triangle on the bottom). **Solve:** calculate the scale factor k from a complete column, then multiply by k to go to the target triangle (divide by k to go back); check that the ratios are equal (model L1 in the course notes).

Worked model (2.2, triangles ACE and CBD , similar by two pairs of corresponding angles): $CB = AB - AC = 7,5 - 3 = 4,5$; scale factor from CBD to ACE : $k = \frac{AC}{CB} = \frac{3}{4,5} = \frac{2}{3}$; $AE = \frac{2}{3} \times CD = \frac{2}{3} \times 9 = 6$; $CE = \frac{2}{3} \times BD = \frac{2}{3} \times 5,4 = 3,6$. Check: $\frac{3}{4,5} = \frac{6}{9} = \frac{3,6}{5,4} = \frac{2}{3}$.

Phase 5 (52–55): Cliffhanger

Put the diagram for 2.5 on the board (the butterfly without parallel lines, only one known angle): “Here, no parallel lines. How can we prove that they are still similar? Answer in the next lesson.” Do not solve it.

Homework. Learn: re-read 2.2 and the method (course notes, pp. 14–16). • By heart: the three steps; “opposite the largest angle lies the longest side”. • Evening drills: “Evening of S2” block on the sheet (answers at the bottom of the block). • Do: finish **2.1** (both methods for b), started in class) + **5.1** (Naïm’s work: spotting the matching error at the first available study slot once the method is established) + the butterfly microtest as a self-test (course notes, p. 17). Track M (stop at 35–40 min): drills + end of 2.1 (b written out following the notebook model) + 5.1 a) (identify the skipped step) + the microtest (deliberate scaffolding). Optional Track H, as a replacement: **invent a twin of 2.2** (different values, with its worked solution); if an idea was sketched out in class, take it through to the worked solution, otherwise design the twin from start to finish.

ÉCOLE DIRECTE (school messaging platform): text ready to paste

Read the course notes: model L1 (match, set up, solve) and the worked example.

Learn by heart: the three steps; opposite equal angles lie corresponding sides; opposite the largest angle lies the longest side.

Evening drills: “Evening of S2” block.

Do: finish 2.1, do 5.1 and the quick-fire questions in the course notes.

Rework: the complete write-up of 2.2 using the model in the notebook.

Optional: invent a twin of 2.2 with different values; finish an idea sketched out in class along with its solution, or design the twin from start to finish.

If running late. The final write-up of 2.2 moves to the start of S3; 1.4 (the butterfly using angles) and questions

a–b of 2.2 cannot be cut.

Success if... exit check “In two similar triangles, the side corresponding to the longest side is...?”: the longest side of the other triangle (opposite the largest angle); a check on matching, not rote recitation.

Teaching note: [S] Research describes, in specific contexts, how fluency, accuracy, worked examples and variation fit together, as well as the role of representations. [T] The timed drill, the 2.2 model solution, the 6.1/6.2 twins and the table of corresponding sides are local adaptations. [D] Remove the timer or lighten the load if matching, representation and justification are not progressing.

S3: Configurations, SAS for similarity, and the vote (format B, 55 min)

Pupil objective. Match sides in tricky configurations (anti-parallel); use the third criterion when only one angle is known; prove by contradiction that two lines are not parallel (2.5 c); settle a true/false question with a counterexample.
Skills: calculating, reasoning, communicating.

Kadai (to write on the board, left zone, boxed). “How can we prove that two triangles are similar when we only know one angle?”: the matome answers this (the 3rd criterion).

Materials. Exercise sheet 2.3, 2.5, 1.5; teacher instructions; mini-whiteboards.

Phase 1 (0–8): Fluency + correction of 2.1

Fluency: 2 quick-fire matchings (left corner). Correction of 2.1: both methods side by side in the left corner (scale factor $k = 1,5$ / cross-multiplication). The 2 min saved on the drill go to the climax (2.5, Phase 3).

Q1. “When should we prefer the scale factor, and when cross-multiplication?”, Goal: learn that choosing a method is a matter of JUDGEMENT. Expected: the scale factor is faster when k is simple ($\frac{3}{2}$, $\frac{2}{3}$); cross-multiplication is always available, even when k is an awkward fraction, and will be used again in the intercept theorem chapter; on your paper, calculate using the scale factor k (step 4 of model L1) and only add cross-multiplication if the question asks for it (course notes, Part 3.1).

Phase 2 (8–22): 2.3, the two configurations (including the anti-parallel trap)

Config 1 (separate triangles) independently, announced correction: only the check of the three ratios is written up, in the left corner. Config 2 (anti-parallel): before any calculation, hold a vote on “Which side is used to calculate DE ?”: the vote brings the error out into the open before it costs marks.

Q2. “Config 2: match the sides, then calculate DE and AE .”, Goal: defuse the trap of matching by letter order. Expected: corresponding sides $[AD] \leftrightarrow [AC]$, $[AE] \leftrightarrow [AB]$, $[DE] \leftrightarrow [CB]$; $k = \frac{4}{10} = \frac{2}{5}$; $DE = 3$, $AE = 3,2$. Error to elicit: $[AD] \leftrightarrow [AB]$ (by position of the letters), have pupils verbalise “I follow the ANGLES, not the order of the letters”. Check: the results are smaller, which is consistent with $k < 1$.

Phase 3 (22–40): 2.5 in depth, SAS then proof by contradiction (the climax)

The third criterion is beyond the required syllabus; this Track R exercise is guided in class, and its progression takes up the CENTRE of the board. Resolving the S2 cliffhanger. Diagram on the board; ask pupils what is missing compared with 1.4: no parallel lines are given, so the “alternate interior angles” reflex is blocked.

Q3. “How many pairs of equal angles do you know here? Which criterion in the course notes only needs one angle?”, Goal: use the 3rd criterion. Expected: only one pair ($\widehat{DAC} = \widehat{BAE}$, vertically opposite angles); **SAS for similarity**, an equal included angle between two proportional sides: $\frac{AB}{AD} = \frac{28}{21} = \frac{4}{3}$ and $\frac{AE}{AC} = \frac{96}{72} = \frac{4}{3} \Rightarrow$ similar. Error to elicit: matching AB with AE (two sides of the same triangle), “I compare one side from EACH triangle, either side of the equal angle”.

Q4. (2.5 c: stepping up) “In 1.4, the ‘wings’ of the butterfly were parallel. Here, are (DC) and (BE) parallel? PROVE it.”, Goal: first proof by contradiction in the chapter—assume, deduce, reach a contradiction. Expected: if $(DC) \parallel (BE)$, then $\widehat{DCA} = \widehat{EBA}$ (alternate interior angles, transversal (BC)); yet similarity from a) gives $\widehat{DCA} = \widehat{AEB}$; so we would need $\widehat{ABE} = \widehat{AEB}$, meaning ABE would be isosceles at A , which is impossible: $AB = 28 \neq 96 = AE$. Therefore they are not parallel. Prompt: “suppose they are parallel: write down EVERYTHING that follows, and look for the contradiction”. Error to elicit: concluding “not parallel, you can see it”, replaying “observing is not proving” (S1). Question b) (the ratio of the areas) is left open on the board: “the lengths are multiplied by $\frac{4}{3}$, so what is the area multiplied by? Answer in the next lesson.” Do not solve it: this hooks into k^2 in S4.

Anticipation. ✗ “we can’t do anything, no parallel lines” (relying solely on the angle criterion gets stuck); ✗ $\frac{AB}{AE}$ (same triangle); ✓ $\frac{4}{3} = \frac{4}{3}$ around the vertically opposite angle; on c): ✗ “it doesn’t look like it on the diagram” (the eye proves nothing); ✓ the impossible isosceles triangle ($28 \neq 96$).

Phase 4 (40–52): 1.5 true/false by vote (Track R level on the test)

Show-of-hands vote BEFORE justification, statement by statement; a numerical counterexample is required for every “false”.

Quick answers (teacher): a) T (60/60/60); b) F (80/50/50 vs 20/80/80); c) F (90/30/60 vs 90/45/45); d) T (90/45/45 forced);

e) F: 80° at the main vertex gives 80/50/50, at the base gives 80/80/20; f) F ($k \neq 1$); g) T ($k = 1$).

Q5. (after e) “What if the angle were 100° instead of 80° ?” Goal: prepare the test Challenge question without saying so. Expected (touch on briefly): an obtuse angle cannot be at the base of an isosceles triangle (twice 100° exceeds 180°), so it must be at the main vertex—only one possible angle distribution, always similar. Say nothing to the pupils about the test.

Phase 5 (52–55): Protected matome

Re-read the kadai, have two pupils state a one-sentence answer, then compare it with the official written summary: when only one angle is known, SAS requires two pairs of proportional sides. These three minutes are ring-fenced for the matome and must not spill over into the vote or the homework.

Differentiation. Fast finishers, Track H: 2.6 (the “areas” twin of 2.3), then an additional numerical counterexample for each “false” in 1.5: 5.1 was already set for homework last night.

Homework. Learn: the 3rd criterion (SAS for similarity, assumed without proof and not required for the Brevet) and the butterfly configuration (course notes, pp. 12–13 and 16–17). • By heart: the sources of equal angles (marked given, common angle, vertically opposite angles, alternate interior / corresponding angles) and the 3rd criterion. • Evening drills: “Evening of S3” block on the sheet (answers at the bottom of the block). • Do: **2.4 a) to d)** (the 3-4-5 triangle; question e) on areas remains the optional extension) + the **3rd-criterion microtest** as a self-test (course notes, p. 13). **Heaviest evening of the chapter:** 2.4 includes a full-scale construction; the 60-minute red line is never crossed (see Part 9). No evening Brevet question tonight: problems **6.1 and 6.2** join the full weekend paper (below). Track M (stop at 35–40 min): drills + 2.4 a) and b), then c) only if the time cap allows + the microtest (deliberate scaffolding). Optional Track H, as a replacement: **Brevet 6.9** (the hidden common angle).

End of week. Alternation [D]: this block is only activated if this weekend is the announced “exam-paper” weekend (at most one weekend in two). Otherwise: 30 min of review/error log or rest; none of the workload below is due or carried over. If this lesson ends the week (default case: S3 for 4 lessons a week; otherwise, move this block to the last lesson of the week): full paper under exam conditions, problems **Brevet 6.1** (skeleton of 2.2) and **6.2** (the butterfly) + **5.3** (write a complete solution following the chapter template), written in one sitting as in the Brevet, without the worked solutions (not yet handed out: correction takes place back in class); then 10 min on the error log (re-read the entries from Chapters 00 and 01 and from the week). Same starting point for all; M may stop at 30 min with a written attempt. On return: final answers displayed at the start of S4; two pieces of work on 6.1 are reviewed against the marking grid. **Weekly quick quiz** (routine carried over from Ch. 00): at the start of the lesson opening the following week (S4), 1.3 b) used as the reference structure, with possibly altered values (5 min, formative): it replaces the opening fluency drill that day. (The past week was covered by the quick quiz carried over from Ch. 01: Ch. 01 exercise sheet, exercise 5.12, taken at the start of the chapter.)

ÉCOLE DIRECTE (school messaging platform): text ready to paste

Read the course notes: SAS for similarity; then pp. 16–17: the butterfly configuration.

Learn by heart: marked given, common angle, vertically opposite angles, alternate interior or corresponding angles, calculated third angle; know SAS for similarity.

Evening drills: “Evening of S3” block.

Do: 2.4 a) to d), then the quick-fire questions in the course notes.

Do: the full-scale construction in 2.4 b), using a ruler.

Optional: Brevet 9, “the hidden common angle”, to finish if started in class, or to explore otherwise.

Exam-paper weekend option (to paste if designated by the rotation).

ÉCOLE DIRECTE (school messaging platform): text ready to paste

Do at the end of the week: Brevet questions 1 and 2, then 5.3, written in one sitting as in the Brevet with worked solutions closed.

Rework: one entry in the error log after the in-class correction.

Light weekend option.

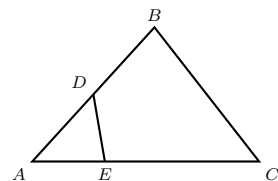
IDEAL BOARD (final state): to project or reproduce

LEFT: drill corner + the 2.3 trap + the vote

Kadai: "Similar with only one known angle?"

Drill corner: 2.1: two methods side by side ($k = 1,5$ / cross-multiplication).

2.3 config 2: $\widehat{ADE} = \widehat{ACB}$; $AD = 4$, $AC = 10$, $AB = 8$, $CB = 7,5$.



✗ $[AD] \leftrightarrow [AB]$ (the letters): crossed out

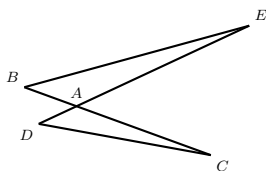
✓ $[AD] \leftrightarrow [AC]$, $[DE] \leftrightarrow [CB]$; $k = \frac{2}{5}$; $DE = 3$, $AE = 3,2$

1.5 vote: a T · b F · c F · d T · e F · f F · g T

e) 80° : at the main vertex (80/50/50) or at the base (80/80/20)

CENTRE: 2.5, the butterfly without parallels (the climax)

(BC) and (DE) intersect at A ; $AB = 28$, $AC = 72$, $AD = 21$, $AE = 96$; no parallel lines given.



✗ "nothing we can do: no parallel lines" (the alternate-interior reflex is blocked)

✗ matching AB with AE (two sides of the same triangle): crossed out

a) ✓ vertically opposite at A ; $\frac{AB}{AD} = \frac{28}{21} = \frac{4}{3} = \frac{96}{72} = \frac{AE}{AC}$: **SAS for similarity** $\Rightarrow DAC$ and BAE similar ($D \leftrightarrow B$, $A \leftrightarrow A$, $C \leftrightarrow E$)

c) **by contradiction:** if $(DC) \parallel (BE)$, then $\widehat{DCA} = \widehat{EBA}$ (alternate interior); yet (a): $\widehat{DCA} = \widehat{AEB}$; ABE would be isosceles at A , impossible: $AB = 28 \neq 96 = AE$. Not parallel. ■

b) **left open:** lengths $\times \frac{4}{3}$, and the area, multiplied by what? $\rightarrow$ next lesson (k^2).

RIGHT: summary (written summary)

Match using the ANGLES, never the position of the letters.

3rd criterion (SAS for similarity): an equal included angle between two proportional sides $\Rightarrow$ similar.

Reasoning by contradiction: assume, deduce, reach a contradiction, so the assumption was false.

The largest angle is opposite the longest side.

✂ TO COPY INTO THE NOTEBOOK (word for word)

Matching is done using the ANGLES, never the order of the letters: opposite equal angles lie corresponding sides.

Third criterion ("SAS for similarity"), assumed without proof: if two triangles have an equal angle INCLUDED between two pairs of proportional sides, then they are similar. (Useful when only one angle and side lengths are known. This is an extension to the course notes: it is not required for the Brevet.)

Reasoning by contradiction (seen on the butterfly without parallel lines): to show that two lines are not parallel, we assume that they are, work out the consequences, and reach a contradiction. Here: if $(DC) \parallel (BE)$, the alternate interior angles give $\widehat{DCA} = \widehat{EBA}$; similarity gives $\widehat{DCA} = \widehat{AEB}$; therefore $\widehat{EBA} = \widehat{AEB}$: ABE is isosceles at A , so $AB = AE$; yet $AB = 28$ and $AE = 96$. The assumption was therefore false.

If running late. Limit the vote on 1.5 to items e) and g). Configuration 2 of 2.3, SAS in 2.5 a), the proof by contradiction in 2.5 c) and the three protected minutes for the matome cannot be cut: they form the core mathematical thread of the lesson.

Success if... on the re-vote for 1.5 e), record the before/after distribution, the number of pupils citing both angle distributions for 80° and the number correctly completing the mini-whiteboard sentence; follow-up depends on these observations.

Teaching note: Vote before justification on 2.3 and 1.5: surface misconceptions in order to address them, numerical counter-example required. Changing configurations around a single method (butterfly $\rightarrow$ anti-parallel $\rightarrow$ SAS): bianshi variation. Matching by letter order is elicited and then ruled out by the class. The proof by contradiction in 2.5 c) extends "measuring is not proving" (S1): we conclude neither by eye nor by measurement, we prove.

S4: Enlargement and reduction, k , k^2 , k^3 (format A, 55 min)

Pupil objective. Conjecture the general rule for areas, prove it on a rectangle, then know the k , k^2 , k^3 effects and use them both ways. **Skills:** investigating, reasoning, calculating.

Kadai (to write on the board, left zone, boxed). “When a shape is enlarged by scale factor k , what are its lengths, areas and volumes multiplied by?": the matome answers this (k , k^2 , k^3).

Materials. Course notes Part 4; exercise sheet 3.2 (activity), 3.1 (drill), 3.3; mini-whiteboards; ruler.

Phase 1 (0–10): Quick quiz + differentiated weekend review

Week 2 retrieval quick quiz: 1.3 b) used as the reference structure, with possibly altered values, 5 min, formative. If the “exam-paper” weekend was activated, display the answers to 6.1/6.2/5.3 and compare two pieces of work on 6.1; otherwise, compare two anonymised error-log entries on matching sides. The marking grid remains the same.

Q1. “Would both pieces of work get full marks? What separates them?”, Goal: the marking grid as a self-assessment tool. Expected: the difference lies in the written presentation (the three steps written out, the check), rarely in the raw answer.

Phase 2 (10–24): Activity 3.2, conjecturing k and k^2

The quadrilateral on a grid. Quick construction in a) (grid). The core: c) $\rightarrow$ e), the conjecture written down by the pupils. Do not give k^2 away in advance.

Q2. “The area of ABD is 12. Its reduction by scale factor 0,5 has area...? Complete: $\mathcal{A}' = 0,5 \cdots \times \mathcal{A}$.”, Goal: bring out the exponent 2. Expected: $\mathcal{A}' = 3$, and $3 = 12 \times 0,25 = 12 \times 0,5^2$. Prompt: “ $3 = 12 \times$ what? and 0,25 is 0,5 to the power of what?”.

Anticipation. ✗ “area $\times 0,5$ ” (the $\times k$ error), display it and confront it with the grid count ($3 \neq 6$); ✗ “the angles change too” (no: preserved); ✓ “area $\times 0,5^2$ ”; ✓ “and scale factor 1,2 will give $\times 1,2^2$ ”.

Phase 3 (24–34): Proof of k^2 on a rectangle (matome) + institutionalisation

Proof on the board: rectangle $L \times \ell$ enlarged by scale factor $k \rightarrow (kL) \times (k\ell) = k^2 \mathcal{A}$; then the square of side 1 enlarged $\times 4$ contains $16 = 4^2$ grid squares.

Q3. “Why $\times k^2$ for the area, and not $\times k$?”, Goal: link the exponent to the number of dimensions. Expected: an area has two dimensions; each is multiplied by k , so the area is multiplied by $k \times k = k^2$. Prompt: “how many dimensions does a length have? an area? a volume?” $\Rightarrow k, k^2, k^3$. Close the open question from S3 (2.5 b): the butterfly without parallel lines had its lengths multiplied by $\frac{4}{3}$, so its area is multiplied by $(\frac{4}{3})^2 = \frac{16}{9}$, and BAE (the enlargement) has the larger area. One line on the board: the loop is closed.

TO COPY INTO THE NOTEBOOK (word for word)

Effects of an enlargement or reduction of scale factor k . Lengths (sides, perimeters) are multiplied by k ; angles are preserved (the shape does not change); areas by k^2 ; volumes by k^3 .

Why k^2 : for a rectangle of length L , width ℓ and area $\mathcal{A} = L\ell$, the enlarged rectangle has area $\mathcal{A}' = (kL) \times (k\ell) = k^2 \times (L\ell) = k^2 \mathcal{A}$ (two dimensions).

Working backwards: if we know the ratio of the areas, it equals k^2 ; we find k by looking for the positive number whose square is equal to this ratio.

Phase 4 (34–44): Timed drill 3.1 (8 min)

8 min, no calculator; f) and g) as multiple-choice questions by vote. Quick correction.

Immediate differentiation. Pupils needing support: **Application 3.1 bis (M)** “the photographed pennant” following 3.1 on the exercise sheet (simple scale factor, sense check required): a stepping stone before 3.3. Fast finishers, Track H: 3.7 (doubling a cube).

Quick answers (teacher): a) $26 \times 0,25 = 6,5$ cm b) 100 cm² c) $27\,000 \div 30^3 = 1$ m³ d) $k = 3$ e) 125 f) $\frac{25}{9}$ g) $\frac{3}{4}$ h) trap: angles are preserved, not multiplied.

Phase 5 (44–55): 3.3 working backwards (test level)

“ k^2 first, k second.”

Q4. “Question a: justify similarity before calculating the area.”, Goal: never calculate an area without first establishing similarity. Expected: $\widehat{C}$ common + corresponding angles $((AB) \parallel (A'B')) \Rightarrow$ similar; $k = \frac{2}{1,5} = \frac{4}{3}$; area $= 2,7 \times \left(\frac{4}{3}\right)^2 = 2,7 \times \frac{16}{9} = 4,8 \text{ cm}^2$. Prompt: “what is the ratio of the lengths before touching the areas?”. Then b) $k^2 = 25$ and, since the scale factor k is positive, $k = 5$; c) multiple-choice $24 \div 1,25^2 = 15,36$; d) $8 \times 3000^2 = 7,2 \times 10^7 \text{ cm}^2 = 7200 \text{ m}^2$ (plausible: a football pitch $\approx 7000 \text{ m}^2$).

Differentiation. Fast finishers: 6.3 (similar triangles and areas) independently.

Homework. Learn: the table of effects and the proof of k^2 (course notes, pp. 19–20). • By heart: the four effects (k , angles, k^2 , k^3) and “an area is multiplied by k^2 ; for $k \neq 1$, this factor differs from k ”. • Evening drills: “Evening of S4” block on the sheet (answers at the bottom of the block). • Do: finish **3.3** (working backwards) + the effects microtest as a self-test (course notes, p. 21) + **the evening Brevet question: problem 6.3** (similar triangles and areas: today’s k^2 in Brevet format). (Charter recalibration: 5.3 and 6.2 moved to the full weekend paper; 6.3 is brought forward from S5 to tonight.) Announce: 3.3 b)–d) is the candidate for the quick quiz carried over into Week 1 of Chapter 01B: anyone who writes it out tonight already knows the format and the assessment criterion. Track M (stop at 35–40 min): drills + 3.3 b) and c) (with the table of effects copied at the top of the page) + the microtest + Brevet 6.3, completed with model A2 from the course notes (p. 21) in front of them (deliberate scaffolding). Track R (40–50 min): also do 3.3 d) and write up Brevet 6.3 without the model. Optional Track H, as a replacement: **3.7** (doubling a cube: the Delian problem).

ÉCOLE DIRECTE (school messaging platform): text ready to paste

Read the course notes: the effects on lengths, angles, areas and volumes, then the proof for the square of the scale factor.

Learn by heart: lengths and perimeters multiplied by k , angles preserved, areas multiplied by k^2 , volumes multiplied by k^3 .

Evening drills: “Evening of S4” block.

Do: finish 3.3, do the quick-fire questions in the course notes and Brevet 3, with worked solutions closed.

Optional: 3.7, “doubling a cube”: to finish if started in class, or to explore otherwise.

Quick quiz in the next chapter: 3.3 b) to d) revisited in a very similar form.

IDEAL BOARD (final state): to project or reproduce

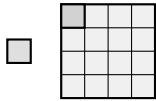
LEFT: the conjecture (3.2)

Kadai: “Lengths, areas, volumes: multiplied by what?”

Quadrilateral on a grid, diagonal $[BD]$; area $ABD = 12$.

Reduction $\times 0,5$: area =?

Seen on a 4×4 square:



side 1, area 1 $\rightarrow$ side 4, area $16 = 4^2$

CENTRE: the proof

✗ “area $\times 0,5$ ”: false ($3 \neq 6$)

✓ $\mathcal{A}' = 3 = 12 \times 0,25 = 12 \times 0,5^2$

Proof (rectangle):

$\mathcal{A}' = (kL)(k\ell) = k \times k \times L\ell$
 $= k^2 \mathcal{A}$ (two dimensions)

1 dimension $\rightarrow k$; 2 $\rightarrow k^2$; 3 $\rightarrow k^3$

S3 open question resolved: 2.5 b): lengths $\times \frac{4}{3}$

$\Rightarrow$ areas $\times \left(\frac{4}{3}\right)^2 = \frac{16}{9}$ ✓

RIGHT: summary (written summary)

Scale factor k : lengths $\times k$; angles preserved; **areas** $\times k^2$; **volumes** $\times k^3$.

Red line: the area factor is k^2 (equal to k only if $k = 1$).

Working backwards: ratio of areas $= k^2$, then find $k > 0$.

If running late. Drill 3.1 moves to the opening fluency slot of S5; 3.3 a) (similarity before area) cannot be cut; the proof of k^2 is never cut (candidate for Ex. 4B on the test).

Success if... exit check “area $\times 49$: scale factor? lengths $\times 10$: volume?” ($k = 7$; volume $\times 1000$): two $k/k^2/k^3$ traps, not two rote recitations.

Teaching note: [S] Research describes justification, questioning, fluency and the organisation of problem-based lessons in observed classrooms. [T] The conjecture followed by the proof of $\times k^2$, drill 3.1 and the $\times k$ error are local adaptations. [D] The timer is removed if conceptual accuracy drops; the proof and checking by measurement remain the priority.

S5: Critical thinking and the Problem Wheel (format B, 55 min): end of the scheduled R selection

Pupil objective. Create a problem under constraints, have it solved and revise it (the climax of the lesson); invalidate a claim by calculation (critical thinking); tackle a scale problem using the Wheel. **Skills:** search, model, reason, communicate.

Kadai (to write on the board, left zone, boxed). “How can we make a new problem from a solved problem (and spot what we changed)?”: the matome answers this (modify / create / have solved).

Materials. Exercise sheet 3.4, 3.5, 5.2, 5.3, 4.1; teacher instructions; Wheel poster; worked solutions for the exercise sheet (to hand out).

Phase 1 (0–10): Fluency + correction + “checking without calculating”

Quick correction of the homework (3.3, Brevet 6.3 — from the brevet, French national exam at the end of Year 10). Then 5.2 orally (one sentence per item, no written calculations allowed).

Quick answers (teacher): 5.2: a) $3^2 = 9$, not 6; b) a reduction shortens ($16 > 8$ impossible); c) impossible (the longest side is opposite the largest angle); d) $\text{area} \div 100^2 = 10\,000$, not 100.

Q1. “How do we know a result is wrong without recalculating it?”, Goal: activate order of magnitude / consistency as a detector. Expected: by plausibility (direction of k , dimension, largest angle $\rightarrow$ longest side), the habit to perform before handing in any paper. Then **3.4 as a quick flash** (2 min, orally, the easy part is pushed out of the main lesson body in favour of the climax, it lives in the top-left corner): is AMN a reduction of ABC ? yes, $k = \frac{2}{6} = \frac{1}{3}$, $\text{area} = 9 \times \frac{1}{9} = 1 \text{ cm}^2$; verbalised check without recalculating: consistent, 1 is 9 times smaller than 9, since $k^2 = \frac{1}{9}$.

Phase 2 (10–25): Problem posing from 5.3 (the climax)

Korean problem posing

Have pupils modify OC in problem 5.3 and predict the effect on the scale factor and the two required lengths. Pairs then create two similar triangles, the second being a reduction of the first with scale factor $\frac{3}{4}$, with one unknown in each, without any triple provided by the teacher. They swap their problems, reconstruct the matching, solve them and point out any ambiguities. Whole-class discussion: compare two choices of data, then have each problem and its check revised.

The whole-class discussion is staged in the CENTRE of the board: the solved model 5.3, the modification predicted before calculation, then two pair problems copied out, compared and revised: this is the climax of the lesson, going beyond the exercise.

Q2. “Before any calculation: if OC goes from 4 to 3, what happens to k , OD and AB ?”, Goal: predict the effect of a change before calculating. Expected: $k = \frac{3}{6} = \frac{1}{2}$ (it decreases); so OD decreases and AB increases; check: $OD = 5,4 \times \frac{1}{2} = 2,7$; $AB = 5 \div \frac{1}{2} = 10$. Follow-up: “Is OD in the small or in the large triangle? And AB ?”.

Q3. (on each displayed pair problem) “Is this problem solvable as it stands? What is missing, what is superfluous?”, Goal: bring out the four criteria of a good problem. Expected: SOLVABLE; SUFFICIENT data (we can match sides without guessing: the vertex correspondence is written down); CONSISTENT data (no length contradicts the scale factor $\frac{3}{4}$); possible CHECK (the three ratios). Follow-up: “can you solve it without asking its author a single question?”: if not, we revise the problem, not the reader.

Anticipation (pair problems):

- ✗ vertex correspondence missing (the matching has to be guessed): display it, have pupils put what is missing into words;
- ✗ superfluous datum that contradicts the scale factor $\frac{3}{4}$ (a third side chosen “at random”);
- ✓ minimal solvable problem: two corresponding lengths fixing $k = \frac{3}{4}$, one unknown per triangle;
- ✓ problem with a planned check (all three ratios stated).

Phase 3 (25–28): Introducing the Problem Wheel

Introduce the Wheel (KNOW/WANT/TOOLS/CHECK): it will be used on 4.1 and in groups from the next lesson onwards. (3.4 was covered as a quick flash in phase 1.)

Phase 4 (28–40): 3.5 the Statue of Liberty (critical thinking, pairs)

a) quick whole-class work: $k = \frac{11.5}{46} = \frac{1}{4}$. b) in pairs, 8 min.

Q4. “What assumptions would allow us to multiply the mass by k^3 ?” Expected: same materials, same densities, reduction of all internal dimensions. With the conventional data in the exercise, the model predicts $225/64 \approx 3,5$ t; 14 t contradicts it. The outer shape alone remains undecidable. Do not invent an explanation about plate thickness or manufacturing.

Anticipation. ✗ $225 \div 4 \approx 56$ t (k); ✗ $225 \div 4^2$ (k^2 , mass is not an area); ✓ $225 \times (\frac{1}{4})^3 \approx 3,5$ t $\neq$ 14 t.

Phase 5 (40–52): 4.1 the park to scale, with the Wheel

Go through the 4 quadrants before any calculation; sketch required (question a).

Q5. “What I KNOW? What I WANT? My TOOLS? My CHECK?”: (park data and scale; actual lengths and area; proportionality and k/k^2 effects; plausibility of orders of magnitude). Expected: scale = $\frac{6}{1200} = \frac{1}{200}$; $EF = 4,5$ cm, $DF = 7,5$ cm; right-angled at E ($9^2 + 12^2 = 15^2$); actual area = $\frac{12 \times 9}{2} = 54$ m².

Q6. “Question e: is the area on the map $\frac{1}{200}$ of the actual area?”, Goal: the central trap. Expected: NO, the area is divided by $200^2 = 40\,000$. Two-way check: 54 m² = $540\,000$ cm², $\div 40\,000 = 13,5$ cm²; and directly $\frac{6 \times 4,5}{2} = 13,5$ cm². Both ways agree: point this out.

Phase 6 (52–55): Handing out the worked solutions + announcing the end of R

Handing out the exercise sheet worked solutions. State the four-rule working agreement for using them (see Part 2) and frame it positively: “I am giving this to you because I am asking you to work more, not less; it makes you independent, provided you try on your own first.”

IDEAL BOARD (final state): to project or reproduce

LEFT: drill corner + critical thinking + park

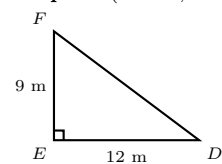
Kadai: “Creating a problem from a solved problem”

Drill corner: 5.2 without calculating: $3^2 = 9$ (not 6); a reduction shortens; large angle $\leftrightarrow$ long side; area $\div 100^2$. 3.4 : $k = \frac{1}{3}$, area 1 cm² (check: $k^2 = \frac{1}{9}$).

3.5 statue, model: $k = \frac{1}{4}$; same materials and internal geometry assumed: $225/64 \approx 3,5$ t $\neq$ 14 t: full model refuted, outer shape undecidable.

✗ $225 \div 4 \approx 56$ t (k instead of k^3): crossed out

4.1 park (sketch, Wheel):



scale $\frac{6}{1200} = \frac{1}{200}$; actual area 54 m²; e) area $\div 200^2 = 40\,000$: $13,5$ cm² (two ways)

CENTRE: the “modify, create, have solved” cycle (the climax)

5.3, the model: vertically opposite at O + $\widehat{OAB} = \widehat{OCD} \Rightarrow$ similar; $k = \frac{4}{6} = \frac{2}{3}$; $OD = 3,6$; $AB = 7,5$

Modify ($OC : 4 \rightarrow 3$), predicted before calculation: k decreases $\Rightarrow OD$ decreases, AB increases; checked: $k = \frac{1}{2}$; $OD = 2,7$; $AB = 10$

Create (the second triangle is the reduction of the first with scale factor $\frac{3}{4}$; one unknown per triangle), two pair problems displayed, compared:

✗ vertex correspondence missing (guessed): to revise

✗ one datum contradicts the scale factor $\frac{3}{4}$: to revise

✓ solved by a peer, checked (three ratios, large/small consistency) $\rightarrow$ the revised problem is adopted

RIGHT: summary (written summary)

A calculation can invalidate a claim: that is critical thinking.

The Wheel: what I KNOW / what I WANT / my TOOLS / my CHECK, no calculation before TOOLS.

A good problem: solvable · sufficient data · consistent · checkable.

Reduction % do not carry across dimensions (area $\times k^2$, volume $\times k^3$).

 TO COPY INTO THE NOTEBOOK (word for word)

Critical thinking: with the conventional data of exercise 3.5, $k = 1/4$. Assuming identical materials and similar internal geometry, the mass would be $225/64 \approx 3,5$ t. The value 14 t refutes this full model; it is not enough to refute the similarity of the outer shapes alone.

The Problem Wheel (to stick in, it will be used all year): 1. what I KNOW 2. what I WANT 3. my TOOLS 4. my CHECK.

A good problem (to create one): it is solvable; its data are sufficient (the vertex correspondence is written down) and consistent; it can be checked (all three ratios).

A scale of $\frac{1}{n}$ divides lengths by n , areas by n^2 , volumes by n^3 .

Homework (end of the scheduled R selection). Learn: re-read 3.5 and the glued-in Wheel. • By heart: the four questions on the Wheel. • Gammes du soir (evening drills): “S5 evening” block on the sheet (answers at the bottom of the block). • Do: finish 4.1 then move on to 4.2 up to your track’s time limit; any unfinished work is neither due nor carried over. Track M (stop at 35–40 min): evening drills + end of 4.1 + 4.2 a) and b), with the scale model example from the course notes in front of you. Track R (40–50 min): continue 4.2 as far as the time limit allows. Voluntary track H, as a replacement: **4.4** (the two poles) *or* **4.5** (the cascade of altitudes, on the crossed-altitudes configuration in the course notes, p. 17–18) *or* the crossed-altitudes micro-test (course notes, p. 18).

ÉCOLE DIRECTE (school messaging platform): text ready to paste

Read the course notes: the scale model, the reverse direction and the consistency checks.

Learn by heart: what I know, what I want, possible tools, final check.

Evening drills: “S5 evening” block.

Do: finish 4.1, then 4.2 a) and b).

Rework: the critical thinking task on the Statue of Liberty using the Problem Wheel.

Optional: 4.4, “the two poles”.

Exercise sheet worked solutions: try and write out your attempt first, open them afterwards, compare and identify your mistake.

If running late. 4.1 e) moves to the opening flash of S6; revising the created problems can be finished at the start of S6; the core of the climax (predicted modification + creation + one problem tested against the four criteria), 3.5 (critical thinking) and introducing the Wheel cannot be cut. (3.4 already lives as a quick flash in phase 1.)

Success if... on a final item (“area $\times 16$: scale factor?”), record how many answer $k = 4$ without help, how many spot and correct the 56 t error, and how many pairs revise their problem according to the four criteria; these numbers guide the follow-up review.

Teaching note: The modify $\rightarrow$ create $\rightarrow$ solve $\rightarrow$ revise cycle is guided problem posing: knowing how to build the exercise means having gone beyond the exercise, hence its place in the centre of the board. Critical thinking confronts a model with reality (statue, park): fighting against the “suspension of sense-making”. The Wheel = scaffolded metacognition, the teacher’s voice becomes an inner voice (the “inner teacher”). Worked solutions handed out under a working agreement: independence is built with the worked solutions.

S6: Scratch: predicting, drawing and justifying similarity (55 min)

Pupil objective. Program two similar triangles, distinguish between interior angle and turn angle, justify the scale factors on lengths and areas; reinvest a question from the brevet (French national exam at the end of Year 10).

Materials. Exercise sheet 3.8 and 6.4; Scratch with Pen extension; one computer per pair; role swapping; save folder; Memo.

Phase 1 (0–10): Review of 4.2

Compare the scale model and actual size: $540,8 \times 125^2 = 8\,450\,000 \text{ cm}^2 = 845 \text{ m}^2$. Correct an actual dimension error from the class. Do not add a second problem.

Phase 2 (10–14): Predicting without running

Exercise sheet 3.8 a): have pupils complete the loop on their own. Ask why 60° or 120° , without letting the software settle it. Gather justifications.

Phase 3 (14–30): Programming and running

3.8 b)–c): first pupil at the keyboard for the side of 60, second for the side of 90. Check that the shape closes, the pen is lifted between figures and the starting direction is the same. Each pupil keeps the program or screenshot and records their contribution.

Phase 4 (30–40): Comparing and justifying

3.8 d)–e): $k = 1,5$, perimeters 180 and 270, ratio of areas 2,25. Require equal angles to justify similarity. With 30 steps: $k = 1/2$, perimeter 90, area divided by 4. Compare prediction and result; a screenshot does not replace these reasons.

Phase 5 (40–50): Transfer to the brevet

Exercise sheet 6.4 written up individually: angles, correspondence, ratios, unit and check. Correct only the key steps.

Phase 6 (50–55): Matome and test

Have pupils state the rule for changing size, then announce the test: recognise, justify, calculate, k , k^2 , k^3 effects, common proof. Hand out the Memo. Problem 3.6 on cones remains available in the R bank; it is not added to this lesson.

IDEAL BOARD (final state): to project or reproduce

LEFT: question and data

How can we draw two triangles of the same shape by changing a variable?
Sides: 60 then 90 steps. Same loop and same turns.

CENTRE: trials and comparison

Loop: repeat 3 times, move **side** steps, turn right 120° .
Interior angle: $180^\circ - 120^\circ = 60^\circ$.
Each sprite draws three equal sides. Pen up to change starting point.

RIGHT: justified summary

Equal angles: similar triangles.
 $k = 90/60 = 1,5$.
Perimeters: 180 and 270 steps.
Areas: factor $k^2 = 2,25$.
With 30 steps: $k = 1/2$, area divided by 4.

📎 TO COPY INTO THE NOTEBOOK (word for word)

Repeating three times with 120° turns draws an equilateral triangle. Multiplying the side by $k > 0$ keeps the angles at 60° , multiplies the perimeter by k and the area by k^2 .

Homework (leading up to the test). Learn: the Memo, using active recall (cover each box, reproduce it, check); redo the Proof and the k^2 proof; redo missed questions from the micro-tests. • By heart: the definition and characterisation (two angles are enough), the match/set up/solve method, the $k/k^2/k^3$ effects, the list of pitfalls. • Gammes du soir (evening drills): “S6 evening” block on the sheet (if there is a second evening before the test: redo missed questions from the sheet). • Do: the multiple-choice quiz + **twin problem 6.6** (the model car: same

structure as 4.2, corrected this morning); if there is a second evening before the test: add **twin problem 6.5** (the flat floor plan). Track M (stop at 35–40 min): evening drills + the Memo + the quiz + one of the two twin problems of your choice, having first identified the structure of 4.2/2.2. Track R (40–50 min): add **6.5**. Voluntary track H, as a replacement: **6.7** (perimeter $\times k$ versus area $\times k^2$), to finish if started in class, or to discover otherwise.

ÉCOLE DIRECTE (school messaging platform): text ready to paste

Actively read the Memo, then do the multiple-choice quiz in the course notes.

Learn by heart: definition, two angles are enough, match, set up, solve, then the k , k^2 and k^3 effects and the pitfalls in the Memo.

Evening drills: “S6 evening” block; then redo any missed questions.

Do: the quiz and Brevet 6, with worked solutions closed.

Rework: the two proofs, then the missed flash questions from the course notes.

Optional: Brevet 7, “the two trays”, to finish if started in class, or to discover otherwise.

Test in the next lesson: calculator allowed; do not start any new topic.

If running late. Cut back on bank exercises; keep the hands-on activity or shared construction, the justification and the exit ticket. Plan a targeted review for pupils who have not finished.

Observable success. Each pupil keeps their work and explains the method in their own words. Note down follow-up needs without inventing pupil responses.

S7: Test (50 min in a 55-min slot)

Administration. Current paper V3 (5 exercises, 20 marks, mark scheme shown: checked total $4+5+4+5+2=20$, no giveaways). Calculators are allowed on the current paper; any decision to the contrary requires modifying the paper before printing. Slot breakdown: 0–2 min settling in, 2–52 min test, 52–55 min final check and collection. Single starting instruction: “read the whole paper before you begin; the Proof exercise (choice of A or B) carries many marks and requires care”.

“Proofs” variant (45 min). The paper `Controle_Demonstrations` (V1, in `_ANNEXES`) replaces paper V3 when one wishes to assess proof-writing alone: configuration of the two altitudes of an acute-angled triangle, five tools provided (definition, angle characterisation, two angles are enough, angle sum, vertically opposite angles), two properties to prove out of three ($AE \times AC = AF \times AB$, $HB \times HE = HC \times HF$, $EA \times EC = EB \times EH$), mark scheme $9+9+2=20$. No length is calculated in it: it only measures codes D and U; use it as a supplement (graded take-home assignment or second short test), never as the sole test for the chapter, otherwise codes R, A and K remain unassessed.

During the test. Note which questions cause widespread difficulty (material for the feedback session); note who finishes very early (candidates for presentations 7.8 / 7.9 later on).

Marking papers (analysis grid). Code each error: **R** recognise (third angle, all three ratios); **A** match / calculate (corresponding sides, table, k); **K** effects (k vs k^2 vs k^3 , reverse direction); **D** proof / written reasoning (criterion cited without its three elements, missing steps); **U** units / plausibility. The tally by code determines the fluency menu for Chapter 01B.

Returning papers (first 10 minutes of the next lesson, non-negotiable): hand back papers; put ONE anonymised real error on the board, have pupils rephrase and generalise it; require every pupil to correct their paper, recorded in the error log; announce the 2–3 dominant codes and their review schedule. Re-run the vote on 1.5 e) and Ex. 5 (100° challenge) side by side: this pair of items anchors “I test all possible positions of the angle” (see teacher’s note on the test).

Kadai for returning papers (not shown during the test). Where does the first breakdown occur in this script, how can we fix it and what rule will prevent the error from happening again?

IDEAL BOARD (final state): to project or reproduce

LEFT: the real error

Kadai: Where does the first breakdown occur in this script, how can we fix it and what rule will prevent the error from happening again?

[copy here, verbatim and anonymised, the most frequent error from the scripts: only one]

Dominant codes from the test:

R ____ A ____ K ____ D ____ U ____

CENTRE: whole-class treatment

“Is this first line right? Why?”: rephrased by two pupils, never by the teacher

Generalisation, written down: “in what family of figures could this error come back?”

1.5e vs Ex. 5 replayed side by side: 80° (two positions) versus 100° (obtuse → only one)

Review: the dominant codes feed into the fluency drills of Chapter 01B

RIGHT: error log

Entry format, displayed:

1. the problem in one line
2. MY error
3. the full correction
4. “I made a mistake because...”

Re-read before every test.

End of the week. Alternation [D]: this block is only activated if this weekend is the announced “exam-paper” weekend (at most one in two). Otherwise: 30 min of review/error log or rest; none of the workload below is due or carried over. The test closes the week (default case: S7 in the last slot; otherwise, move this block to the last actual lesson of the week). In the first available study slot: well-earned rest (the evening drills sheet says so: re-read the error log, nothing else). At the weekend, two stages: first, correcting the test in the error log (homework no. 1, required below), then the full problem under exam conditions (**7.1** (Geography: reading a map) scale, areas and powers of 10: it reactivates Ch. 01 and feeds into the Brevet booklet) or, alternatively, **7.2** (Biology/Earth Sciences: under the microscope, same core skill, standard form (scientific notation) from Ch. 01), written in one sitting, worked solutions closed (handed out in S5: check AFTER writing), followed by 10 min on the error log. Same starting point; M may stop at 30 min with a written record.

Flash quiz the following week (to announce when returning papers). Exercise **3.3 b)–d)** (reverse direction: finding k , set as evening work in S4, announced as a candidate that evening) will serve as the reference structure, with data possibly changed, formative by default; any grading is announced in advance depending on the goal, with criteria and an opportunity for a retake, at the start of a lesson in the first week of Chapter 01B (Algebra): spaced reactivation bridges across chapters, just as 5.12 from Ch. 01 opened this one.

ÉCOLE DIRECTE (school messaging platform): text ready to paste

Do: when your paper is returned, write a full entry in the error log with the problem, the error, the correction and its cause.

Rework: the priority question and code indicated by the teacher.

Flash quiz in the next chapter: 3.3 b) to d) revisited in a very similar form.

Exam-paper weekend variant (to paste if designated by the rotation).

ÉCOLE DIRECTE (school messaging platform): text ready to paste

Do at the weekend: 7.1, “Geography”, or 7.2, “Biology/Earth Sciences”, in one sitting and with worked solutions closed.

Rework: the error log entry after writing out your solution.

Light weekend variant.

ÉCOLE DIRECTE (school messaging platform): text ready to paste

Rework at the weekend: the error log entry is enough; do not add anything else.

Teaching note: [S] Research describes, in specific contexts, returning scripts, personal error logs and variation. [T] The guide transposes these into an anonymised error, an explained correction, R/A/K/D/U codes and the $80^\circ/100^\circ$ comparison. [D] Keep these routines only if pupils’ work shows better matching and genuinely explained corrections.

5. Ongoing classroom routines (for display)

1. Timed fluency drill to open the lesson: ALWAYS corrected immediately.
2. **Match before calculating:** never set up a proportionality table, never cross-multiply, until the corresponding sides have been identified using the angles.
3. **“Who did it another way?”** after every solution (constant of proportionality versus cross-multiplication; vertically opposite angles versus alternate interior angles...).
4. **Show-of-hands survey** (standard kit): after each method presented at the board, **“who used this method? raise your hand”**: this publicly validates pupils who did not present, gives a snapshot of the class, and legitimises the follow-up “is there a faster, safer way?”.
5. **“Is the result plausible?”** before any conclusion: are all three ratios equal? is the longest side opposite the largest angle? is the direction of k respected? is the dimension ($k/k^2/k^3$) right?
6. Never “do you understand?” → **“how do you know?”** / “by which property?”.
7. Stuck pupil: redirect the SAME open question to another pupil, then have the first pupil rephrase (no funnelling through micro-questions).
8. Homework is ALWAYS followed up (commented correction or real error from the class): never assign homework without feedback; **Short Gammes du soir (evening drills)** in the chosen homework slot (self-corrected), **Evening brevet** (French national exam at the end of Year 10) in S4 and S6, announced exam-paper weekends (at most one in two) (sheets S3 and S7).
9. The board tells the story of the lesson, and its CENTRE tells the story of the climax (the drill lives in the top-left corner): the labelled figure and the reasoning stay displayed until the summary (only the rough-work zone is wiped); pupils copy the written summary at the end, not as the lesson goes along.

5 bis. Three teaching moves (standard kit)

Three minimally invasive teaching moves, to establish once and for all; they do not alter any document or written summary, only the running of climax lessons.

Displayed kadai (K1). In every climax lesson, write two things on the board (left zone, boxed): the concrete problem of the day (mondai) and, underneath, the general challenge (kadai); the “Kadai” line now appears at the top of every lesson sheet and every IDEAL BOARD. The matome must answer the kadai: this is what distinguishes a lesson from which pupils remember “the answer is 4,5” from a lesson from which they remember “similarity is used to measure the inaccessible”.

Co-constructed matome (K2, 3 min): before reading the booklet. The final written summary remains IDENTICAL (pre-written, copied word for word); only how it is generated changes. Three steps: **1)** “What was our question of the day?”: re-read the kadai (left zone). **2)** “State the answer in one sentence”: two pupils make a suggestion, the class refines it, the teacher writes the sentence using the pupils’ words in the right zone. **3)** Then open the booklet: “what makes the definition in the booklet more precise than ours?”, and copy the official written summary, UNCHANGED. Comparing pupil phrasing with expert phrasing is itself a learning experience (status of vocabulary, Part 7). Cost ≈ 3 min, taken from consolidation. (See sheet S0, phase 6.)

Show-of-hands survey (K4). See routine in Part 5: after each presentation, “who used this method? raise your hand”: the snapshot of the class that validates those who did not present and legitimises “is there a faster way?”.

K3: Pre-coded seating plan (kikan-shidō), the circulation tool. Before each climax lesson, print the seating plan and copy onto it the key for the anticipated pupil work from the lesson sheet (“Anticipated pupil work” section): **A** = naive approach, **B** = intermediate, **C** = expert, **X1** / **X2** = the two target errors. During individual or pair work, tick on the plan who produces what (the decision is quick because it is prepared in advance). Benefits: (a) deliberately select the order of presentations for the neriage (from most naive to most expert); (b) know whom to question first; (c) keep a written record of the kikan-shidō (the logbook currently records errors only after the fact). Rationing help: only prompt a pupil who is genuinely stuck; if many stumble over the wording of the problem, pause and clarify for the whole class; otherwise, let them struggle, as a pupil who has not found the answer still has a role in the neriage (sharing their trials). (Mentioned in sheet S0, phase 5.)

6. Remediation script: the 6 major errors of the chapter

Ready-to-use focusing prompts (never give the direct correction: ask a question that refers the pupil back to a check they can carry out on their own). They address the six classic errors from the course notes.

The pupil writes / says...	The teacher replies...
“similar, therefore congruent” (or the converse)	“Superimpose them: can you do it? No: they have the same shape, not the same size. What is k ? If it is not 1, they are not congruent.”
matches sides by letter order (AB with $A'B'$ at random)	“Which angle is opposite $[AB]$? And opposite $[A'B']$? Are those angles equal? Corresponding sides are read opposite equal angles, not from the order of the letters.”
multiplies the area by k (40 for a doubled rectangle)	“How many dimensions does an area have? Each one is multiplied by k : what is $k \times k$? Draw the square enlarged $\times 4$ and count the grid squares.”
“one equal angle, therefore similar”	“Two triangles $40^\circ-110^\circ-30^\circ$ and $40^\circ-70^\circ-70^\circ$ both have a 40° angle: are they similar? How many pairs of angles do we need?”
uses k the wrong way round (multiplies instead of dividing)	“Is this side in the large or the small triangle? Your result must be larger if $k > 1$, smaller if $k < 1$. Write down which triangle k goes from and to.”
concludes “by eye” from the drawing	“Is the drawing necessarily to scale? What proves the angles are equal: a marked datum, a property? We never conclude by eye.”

7. Required oral vocabulary (across the lessons)

- superimposable / congruent triangles
- congruence criteria (SSS, SAS, ASA)
- similar triangles
- corresponding sides / vertices
- scale factor (k)
- angle characterisation
- vertically opposite angles
- alternate interior / corresponding angles
- shared angle
- enlargement / reduction
- scale
- conjecture / prove

Rule: from the lesson in which a term is institutionalised onwards, a pupil response that avoids it (“the side opposite”, “the same angle”) is rephrased by the pupil themselves using the exact term (corresponding, vertically opposite angles. . .).

8. Ongoing formative monitoring and assessment

- **Exit flashes** (mini-whiteboard, S0/S1/S3/S4): the success rate determines the opening of the next lesson; below 70 %, the next fluency drill revisits the missed item; above that, we move on. These are traps (the 47/86, “a single angle”, k vs k^2), not rote recall.
- **Weekly reactivation flash** (see Part 2): inherited from Ch. 01 (5.12) to open the chapter; **1.3 b)** to open S4; **3.3 b–d** passed on to Week 1 of Chapter 01B: the chain is never broken.
- **Course notes micro-tests** (self-tests, answers provided): spread across the Homework blocks from S0 to S4 (the crossed-altitudes micro-test in track H of S5); at the start of the next lesson, ask for a show of hands: “who got everything right? who understood their mistake?”: no marks, just a mental note of recurring first names.
- **Timed drills**: write on the board the number of items completed at half-time; the goal is progress between S2 (1.1) and S4 (3.1), not absolute performance.
- **Participation grid**: aim for every pupil to have come to the board at least once by the end of the chapter (drill, writing up a similarity proof, or vote), ticking off as you go.
- **After the test**: the R/A/K/D/U codes (sheet S7) determine the exact content of the fluency drills for the first 3 lessons of Chapter 01B, as spaced review.

9. Pacing and management: margins, priorities, contingencies

Non-negotiable (never cut). The Proof (S1) and the k^2 proof (S4) • the conjecturing activity (S0) • 2.2 written out in full (the model solution) • **2.5 a) and c)** (the peak of S3: SAS + proof by contradiction) • the “modify, create, have a partner solve” cycle (the peak of S5) • the drills marked in class • the votes (2.3, 1.5, cones) • the commented homework review • the structured announcement of the test. The peak of each lesson holds the centre of the board: when time runs short, compress the drill, never the peak.

Pace of exploration. Exploration time is scaled to the size of the question, not the template of the lesson: 6 min individual + 4 min in pairs for the conjecturing activity (S0); building the proof (S1) takes as long as needed—it is the key lesson, never compress it; 10 min in groups for a genuinely rich problem (3.6 in bank R). Giving too long on a short question creates drift. Any time saved ALWAYS goes to the end-of-lesson drill: more volume covered in class.

Using the ideal boards. The nine “IDEAL BOARD” boxes show the FINAL STATE of the lesson, the one captured in the end-of-lesson photograph. They are never projected before the exploration and whole-class discussion: during the lesson they serve as an aide-memoire for the teacher, and the full-page version in TableauxProjection is only shown to check the summary or as a back-up (rule printed on every BanshoKeikaku sheet: “do not pre-write the answer”). Status of the work shown there: these are the ANTICIPATED responses from the “Anticipation” sections (numbered P1–P4 in S2), never real scripts or pupils’ names; on the board, the responses actually collected are labelled C1, C2, ... with no names, and the C → P mapping is filled in AFTERWARDS (BanshoKeikaku convention). Only two boxes await real classroom material and are left blank until the day itself: the anonymised error in S0 phase 1 and the one in the test-feedback table (S7).

Copying time for the written summary: measure before deciding. The seven “to copy” summaries range from 183 to 851 characters (printed text inside the box, spaces included, title excluded). At a copying speed of roughly 70 characters per minute—a plausible order of magnitude in Year 10 from the board, to be checked with the class, and slower when marked angles, fractions and diagrams are involved—copying alone requires, compared with the minutes actually available for institutionalisation:

Lesson	S0	S1	S2	S3	S4	S5	S6
Estimated copying (min)	8,9	11,0	10,9	12,2	7,0	9,8	2,6
Minutes available	≈ 3	8	≈ 4	3	≈ 4	≈ 2	≈ 2

No written summary fits entirely into the minutes available. **S6** comes close (under a minute short); **S1** and **S4** are short by about 3 minutes (in S1, the written summary *is* the proof, and phase 4 is devoted to it entirely). For **S3**, the longest summary, the trade-off has now been made: launching 2.4 is moved out of the slot and minutes 52–55 are ring-fenced for the co-constructed matome; the same timing appears on the S3 BanshoKeikaku sheet; those three minutes make the matome real, without claiming to absorb all the copying. The other lessons (S0, S2, S5) are short by roughly 6 to 8 minutes. To finish a long summary, the options remain: take the minutes from the consolidation that follows, photograph the board and have pupils finish copying at the start of the next lesson, or shorten the summary after timing it in class. Time the copying once before choosing, and never dictate the summary just to save time.

Exercise sheet coverage (to check at a glance that no exercise is left out). COVERED IN CLASS: 1.1, 1.2b, 1.3, 1.4, 1.5, 2.1, 2.2, 2.3, 2.5, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 4.1, 5.2, 5.3 (and the “modify, create, have a partner solve” block), 6.4. CORE EVENING PRACTICE: 1.1, 1.2, 1.3, 2.1, 2.4, 3.3, 4.1, 4.2, 5.1, 6.3, 6.6. WEEKEND PROBLEMS: 6.1, 6.2, 5.3, then 7.1 or 7.2. EVENING TRACK R: 1.5, 6.5. VOLUNTARY TRACK H, AS A REPLACEMENT: 2.6, 3.7, 4.4, 4.5, 5.4, 6.7, 6.9. FAST-TRACK AND INDEPENDENT BANK: 2.6, 3.7, 4.3, 4.5, 6.7–6.9, 7.2, 7.4–7.10, plus application 3.1 bis (M) “the

photographed pennant” for differentiation in S4. NO EXERCISE ON THE SHEET IS UNUSED; the Part 7 bank remains a resource bank, not an obligation.

Adjustment variables (in order of sacrifice). 1.3 c) (→ homework) • 4.1 e) (→ flash starter in S6) • the second method in 2.1 (→ printed worked solutions) • 2.5 b) (already deferred to the quick review in S4) and, as a last resort, 2.5 c) (→ opening of S4) • 2.4 d–e (→ independent extension) • the entire Part 7 bank (it is a bank, not an obligation). (3.4 now runs as a quick starter in S5, phase 1.)

If the chapter overruns (disrupted start of term, school events...): cut back on bank problems, but keep 3.8 on the photocopier, its justification and the test announcement; spread the S5 review across the warm-up routines; 3.4 and 4.1e move into fluency slots in subsequent lessons (the items are independent). **Never cut S1.**

If a class is ahead (two classes rarely move at the same pace): open the Part 7 bank for independent work at the back of the room from S3 onwards; 7.5 (re-proving the characterisation, SSS case), 7.8 and 7.9 are self-contained and lend themselves to a board presentation, rounded off by 7.10 (comparing three proofs of Pythagoras’ theorem and choosing the most illuminating one); 7.4 (transformations taking one triangle to the other) directly prepares for enlargement in Chapter 06; 7.6 (Sierpiński) and 7.7 (almost twins) are the two challenges.

Struggling pupils identified in S0 (items 5 and 7 of the diagnostic): full M track; see them during support slots using the Memo and the congruent-triangles table as scaffolds; call on them first for the drills (1.1, 3.1) and for calculating the third angle, not for the formal proof. For them, the S1 Proof is told as a story (the “copy the small triangle inside the large one” scheme) before being written out.

Pupils absent for a test or a key lesson. The Memo + micro-tests + multiple-choice questions cover catch-up; if S1 was missed, have a peer explain the proof (“what did we construct inside the large triangle, and why does it work?”).

10. What this guide assumes is in place

Exercise sheet and worked solutions synchronised as of 2026-09-24 (exercises **1.1–7.10** present: 47 numbered exercises, plus application 3.1 bis (M) “the photographed pennant” following 3.1, including the enrichment exercises: 2.5, 2.6, 4.4, 4.5, 5.4, 7.8, 7.9, 7.10; compiled without errors) • teacher’s roadmap on the teacher version of the exercise sheet (☞ prompts at key flashpoints) • audited test V3 (Ex. 4 choice A/B) and its “proofs” variant V1 (45 min, sheet S7) • up-to-date course notes (26 pages; 3rd criterion in Section 2.3, altitudes configuration in Section 3.3 and structural enrichments integrated) • Year 10 instructional model (formats A/B) • Chapter work plan (record of design choices).

Optional enrichments on the exercise sheet (24 September 2026). Six guided blocks extend the exercises without replacing or renumbering them: 1.2 (adding the same length and preserving shape), 2.5 (counterexample to the SAS criterion when the angle is not included), 4.4 (all integer heights for a given crossing height), 6.8 (aspect ratio preserved when cutting), 7.7 (all “almost twin” pairs with scale factor $3/2$ and integer sides), 7.8 (exact construction of a square root using the altitude).

They remain outside the assigned work and the test; the time budget is unchanged. Offer them as H extensions after the original exercise. For 7.7, hide the extension during the initial challenge, as it gives away an example. For 7.8, first establish the relation $AH^2 = BH \times HC$; the extension reuses it for the construction. Block 6.8 rotates the pieces before matching their sides. All necessary concepts are explained within the blocks; no notable identity from chapter 01B is assumed.

Reusable template: this guide follows the structure of the chapter 01 guide; for subsequent chapters, reproduce these headings using the “anticipated errors” section of their Work Plan and the timing audit.